2a. Find an equation for the tangent line to the graph of $y = \tan^3\left(\frac{\pi}{x}\right)$ at the point with $x = 3$.

$$y' = 3\tan^2\left(\frac{\pi}{x}\right)\sec^2\left(\frac{\pi}{x}\right)\cdot\left(-\frac{\pi}{x^2}\right)$$

$$y'\Big|_{x=3} = 3\tan^2\left(\frac{\pi}{3}\right)\sec^2\left(\frac{\pi}{3}\right)\cdot\left(-\frac{\pi}{9}\right) = 3\cdot\sqrt{3}^2\cdot 2^2\cdot\left(-\frac{\pi}{9}\right) = -4\pi$$

$$y\Big|_{x=3} = \tan^3\left(\frac{\pi}{3}\right) = \sqrt{3}^3 = 3\sqrt{3}$$

An equation for the tangent line is:

$$y - 3\sqrt{3} = -4\pi\cdot(x-3)$$

2b. Suppose that a differentiable function f satisfies

$$-||x|-2| < f(x) < |x| \qquad (*)$$

for all x. Show that there is c such that $f'(c) = 0$.

Since f is differentiable, f is continuous.

$(*) \xRightarrow{x=0} -2 < f(0) < 0$

$(*) \xRightarrow{x=-2} 0 < f(-2) < 2$

Since f is continuous on $[-2,0]$ and $f(0) < 0 < f(-2)$, by IVT there is a in $(-2,0)$ such that $f(a)=0$.

Similarly, there is b in $(0,2)$ such that $f(b)=0$.

Since f is continuous on $[a,b]$ and differentiable on (a,b), and $f(a)=0=f(b)$, by Rolle's Theorem there is c in (a,b) such that $f'(c)=0$.