1a. Evaluate the limit $\lim_{x\to 1} \dfrac{x^2+2-\dfrac{12}{x+3}}{x^2+3-\dfrac{12}{x+2}}$.

(Do not use L'Hôpital's Rule!)

$$\lim_{x\to 1} \frac{x^2+2-\frac{12}{x+3}}{x^2+3-\frac{12}{x+2}} = \lim_{x\to 1}\left(\frac{(x^2+2)(x+3)-12}{(x^2+3)\cdot(x+2)-12}\cdot\frac{x+2}{x+3}\right)$$

$$= \lim_{x\to 1} \frac{x^3+3x^2+2x-6}{x^3+2x^2+3x-6}\cdot\frac{1+2}{1+3} = \lim_{x\to 1}\frac{\cancel{(x-1)}\cdot(x^2+4x+6)}{\cancel{(x-1)}\cdot(x^2+3x+6)}\cdot\frac{3}{4}$$

$$= \frac{1^2+4\cdot 1+6}{1^2+3\cdot 1+6}\cdot\frac{3}{4} = \frac{11}{10}\cdot\frac{3}{4} = \frac{33}{40}$$

1b. Suppose that a differentiable function f satisfies

$$f'(x)+3f(x^2)=f(x)^2 \qquad (*)$$

for all $x>0$ and $f(1)=5$. Find $f''(1)$.

$$(*) \xRightarrow{x=1} f'(1)+3f(1)=f(1)^2 \xRightarrow{f(1)=5} f'(1)+3\cdot 5=5^2 \Rightarrow f'(1)=10$$

$$(*) \xRightarrow{d/dx} f''(x)+3f'(x^2)\cdot 2x = 2f(x)f'(x)$$

$$\Downarrow \leftarrow x=1$$

$$f''(1)+6f'(1)=2f(1)f'(1)$$

$$\Downarrow \leftarrow f(1)=5,\ f'(1)=10$$

$$f''(1)+6\cdot 10 = 2\cdot 5\cdot 10$$

$$\Downarrow$$

$$f''(1)=40$$