3. The points P and Q are moving along the graph of a twice-differentiable function $y = f(x)$ in the xy-plane in such a way that their coordinates are differentiable functions of time t, and the tangent line to the graph at the point P intersects the graph also at the point Q at all times. (Assume that the coordinates are measured in meters and the time is measured in seconds.)

Find $f''(2)$ if

① the x-coordinate of Q is -1 and decreasing at a rate of 3 m/s when the x-coordinate of P is 2 and increasing at a rate of 4 m/s,

② $y = 9x - 8$ is an equation for the tangent line to the graph of f at the point with $x = 2$, and

③ $y = -6x - 23$ is an equation for the tangent line to the graph of f at the point with $x = -1$.

Let a and b be the x-coordinates of P and Q, respectively. Then:

$f(b) = f(a) + f'(a)\cdot(b-a)$ at all times

$\Downarrow d/dt$

$f'(b)\dfrac{db}{dt} = f'(a)\dfrac{da}{dt} + f''(a)\dfrac{da}{dt}\cdot(b-a) + f'(a)\cdot\left(\dfrac{db}{dt} - \dfrac{da}{dt}\right)$

$\Downarrow$

$f''(a) = \dfrac{f'(b) - f'(a)}{b-a} \cdot \dfrac{db/dt}{da/dt}$

$\boxed{a=2,\ \dfrac{da}{dt}=4,\ b=-1,\ \dfrac{db}{dt}=-3 \text{ by ①} \quad f'(2)=9 \text{ by ②},\ f'(-1)=-6 \text{ by ③}}$

$\Downarrow$

$f''(2) = \dfrac{-6-9}{-1-2} \cdot \dfrac{-3}{4} = -\dfrac{15}{4}$