

2. Find  $\frac{d^2y}{dx^2}\bigg|_{(x,y)=(1/2,1)}$  if  $y$  is a differentiable function of  $x$  satisfying the equation:

$$\sin(\pi xy) = \frac{1}{x} - \frac{1}{y}$$

$\Downarrow d/dx$

$$\cos(\pi xy) \cdot \pi \cdot (y + xy') = -\frac{1}{x^2} + \frac{1}{y^2} y'$$

$(x, y) = (\frac{1}{2}, 1)$

$$\underbrace{\cos(\frac{\pi}{2})}_0 \cdot \pi \cdot (1 + \frac{1}{2} \cdot y') = -4 + y'$$

$\Downarrow$

$y' = 4$  at  $(x, y) = (\frac{1}{2}, 1)$

$d/dx$

$$-\sin(\pi xy) \cdot (\pi \cdot (y + xy'))^2 + \cos(\pi xy) \cdot \pi \cdot (y' + y' + xy'')$$

$$= \frac{2}{x^3} - \frac{2}{y^3} \cdot (y')^2 + \frac{1}{y^2} \cdot y''$$

$(x, y) = (\frac{1}{2}, 1), y' = 4$

$$-\underbrace{\sin(\frac{\pi}{2})}_1 \cdot (\pi \cdot (1 + \frac{1}{2} \cdot 4))^2 + \underbrace{\cos(\frac{\pi}{2})}_0 \cdot \pi \cdot (4 + 4 + \frac{1}{2} y'')$$

$$= 16 - 2 \cdot 16 + y''$$

$\Downarrow$

$y'' = 16 - 9\pi^2$  at  $(x, y) = (\frac{1}{2}, 1)$