

4. Evaluate the following integrals.

$$\text{a. } \int_0^{\pi/4} \frac{\sec^2 x}{(2 \tan x + 1)^2} dx = \int_1^3 \frac{1}{u^2} \cdot \frac{1}{2} du = \left[-\frac{1}{2u} \right]_1^3 = -\frac{1}{6} + \frac{1}{2} = \frac{1}{3}$$

$$\begin{aligned} u &= 2 \tan x + 1 \\ du &= 2 \sec^2 x dx \end{aligned}$$

$$\text{b. } \int_0^{\pi/4} \frac{\sin x}{(2 \sin x + \cos x)^3} dx = \int_0^{\pi/4} \frac{\tan x \cdot \sec^2 x}{(2 \tan x + 1)^3} dx = \int_1^3 \frac{(u-1)/2}{u^3} \cdot \frac{1}{2} du$$

$$\begin{aligned} u &= 2 \tan x + 1 \\ du &= 2 \sec^2 x dx \end{aligned}$$

$$= \frac{1}{4} \int_1^3 (u^{-2} - u^{-3}) du = \frac{1}{4} \left[\frac{u^{-1}}{-1} - \frac{u^{-2}}{-2} \right]_1^3 = \frac{1}{4} \left(-\frac{1}{3} + \frac{1}{18} + 1 - \frac{1}{2} \right) = \frac{1}{18}$$