4. The points P and Q are moving along the parabola $y = x^2$ in the xy-plane in such a way that their coordinates are differentiable functions of time and the distance between them is constant. (Assume that the coordinates are measured in meters and the time is measured in seconds.)

Determine the rate of change of the distance between the point Q and the origin at the moment when P is at $(2, 4)$, Q is at $(-1, 1)$, and the distance between the point P and the origin is increasing at a rate of 3 m/s.

Let (a, a^2) and (b, b^2) be the coordinates of P and Q, respectively.

$$|OP|^2 = a^2 + a^4 \Rightarrow 2|OP| \cdot \frac{d}{dt}|OP| = (2a + 4a^3) \cdot \frac{da}{dt}$$

$$\Rightarrow 2 \cdot \sqrt{2^2 + 2^4} \cdot 3 = (2 \cdot 2 + 4 \cdot 2^3) \cdot \frac{da}{dt} \Rightarrow \frac{da}{dt} = \frac{\sqrt{5}}{3} \text{ m/s}$$

$$\boxed{a = 2\text{ m}, \ \frac{d}{dt}|OP| = 3\text{ m/s}}$$

$$|PQ|^2 = (a-b)^2 + (a^2 - b^2)^2 \Rightarrow 0 = 2 \cdot (a-b) \cdot \left(\frac{da}{dt} - \frac{db}{dt}\right) + 2(a^2 - b^2) \cdot \left(2a\frac{da}{dt} - 2b\frac{db}{dt}\right)$$

$$\Rightarrow \frac{\sqrt{5}}{3} - \frac{db}{dt} + (2 + (-1)) \cdot \left(2 \cdot 2 \cdot \frac{\sqrt{5}}{3} - 2 \cdot (-1) \cdot \frac{db}{dt}\right) = 0 \Rightarrow \frac{db}{dt} = -\frac{5\sqrt{5}}{3} \text{ m/s}$$

$$\boxed{a = 2\text{ m}, b = -1\text{ m}, \ \frac{da}{dt} = \frac{\sqrt{5}}{3}\text{ m/s}, \ |PQ| = \text{const.}}$$

$$|OQ|^2 = b^2 + b^4 \Rightarrow 2|OQ| \frac{d}{dt}|OQ| = (2b + 4b^3) \cdot \frac{db}{dt}$$

$$\Rightarrow 2 \cdot \sqrt{(-1)^2 + (-1)^4} \cdot \frac{d}{dt}|OQ| = (2 \cdot (-1) + 4 \cdot (-1)^3) \cdot \left(-\frac{5\sqrt{5}}{3}\right) \Rightarrow \frac{d}{dt}|OQ| = 5\sqrt{\frac{5}{2}} \text{ m/s}$$

$$\boxed{b = -1\text{ m}, \ \frac{db}{dt} = -\frac{5\sqrt{5}}{3}\text{ m/s}}$$

The distance between the point Q and the origin is increasing at a rate of $5\sqrt{\frac{5}{2}}$ m/s at that moment.