2a. Let P be the point on the graph of $y = x^5 - x^2$ with $x = 1$. Show that there is a point Q on the graph such that the tangent lines to the graph at the points P and Q are perpendicular to each other.

$y' = 5x^4 - 2x \Rightarrow$ (The slope of the tangent line at P) $= y'|_{x=1} = 3$

We want to show that there is a point Q on the graph such that the slope of the tangent line at Q is $-\frac{1}{3}$.

That is, we want to show that the equation $5x^4 - 2x = -\frac{1}{3}$ (⊛) has a real solution.

Let $f(x) = 15x^4 - 6x + 1$. Then $f(0) = 1 > 0$ and $f(\frac{1}{2}) = -\frac{17}{16} < 0$.

Since f is a polynomial, f is continuous on $[0, \frac{1}{2}]$.

Therefore, by IVT, there is a point c in $(0, \frac{1}{2})$ such that $f(c) = 0$.

Hence the equation ⊛ has a real solution.

2b. In one of the following figures, the graphs of two functions f and g together with their derivatives f' and g' are shown; while in the other, the graphs of two functions h and k together with their second derivatives h'' and k'' are shown. Identify each by filling in the boxes with f, g, f', g', h, k, h'', and k''.