1. Evaluate the following limits by expressing the answers in terms of $A = \lim_{x\to 0} \dfrac{x - \sin x}{x^3}$.
[Do not use L'Hôpital's Rule!]

a. $\lim_{x\to 0} \dfrac{1 - x^2/2 - \cos x}{x^4}$

$$= \lim_{x\to 0} \frac{1 - \cos x - \frac{x^2}{2}}{x^4} = \lim_{x\to 0} \frac{2\sin^2\frac{x}{2} - \frac{x^2}{2}}{x^4}$$

$$= 2\lim_{x\to 0} \frac{\sin^2\frac{x}{2} - \frac{x^2}{4}}{x^4} = 2\cdot \lim_{x\to 0} \frac{\sin\frac{x}{2} - \frac{x}{2}}{x^3} \cdot \lim_{x\to 0} \frac{\sin\frac{x}{2} + \frac{x}{2}}{x}$$

$$= -2\cdot\frac{1}{8}\cdot \underbrace{\lim_{x\to 0} \frac{\frac{x}{2} - \sin\frac{x}{2}}{\left(\frac{x}{2}\right)^3}}_{= A} \cdot \left(\frac{1}{2} \underbrace{\lim_{x\to 0} \frac{\sin\frac{x}{2}}{\frac{x}{2}}}_{= 1} + \frac{1}{2} \right) = -\frac{A}{4}$$

b. $\lim_{x\to 0} \dfrac{x - \tan x}{x^3}$

$$= \lim_{x\to 0} \frac{x - \frac{\sin x}{\cos x}}{x^3} = \lim_{x\to 0} \frac{x\cos x - \sin x}{x^3} \cdot \lim_{x\to 0} \frac{1}{\cos x}$$

$$= \lim_{x\to 0} \frac{x\cos x - x + x - \sin x}{x^3} \cdot 1 = \lim_{x\to 0} \frac{\cos x - 1}{x^2} + \lim_{x\to 0} \frac{x - \sin x}{x^3}$$

$$= \lim_{x\to 0} \frac{-2\sin^2\frac{x}{2}}{x^2} + A = -\frac{1}{2}\Big(\underbrace{\lim_{x\to 0} \frac{\sin\frac{x}{2}}{\frac{x}{2}}}_{= 1} \Big)^2 + A = A - \frac{1}{2}$$