4. The coordinates (x_c, y_c) of the *center* of a region R contained in the first quadrant of the xy-plane are defined by

$$x_c = \frac{W}{2\pi A} \quad \text{and} \quad y_c = \frac{V}{2\pi A}$$

where

- A is the area of R,
- V is the volume of the solid generated by revolving R about the x-axis, and
- W is the volume of the solid generated by revolving R about the y-axis.

Let R be the region lying between the graph of $y = (x^2+1)^{-3/2}$ and the x-axis for $x \geq 0$. Compute one of the coordinates ☑ x_c or ☐ y_c of the center of R.

[Indicate the one you are computing by putting a ✓ in the ☐ to the left of it.]

$$A = \int_0^\infty (x^2+1)^{-3/2}\,dx = \int_0^{\pi/2} (\tan^2\theta+1)^{-3/2}\sec^2\theta\,d\theta = \int_0^{\pi/2} (\sec^2\theta)^{-3/2}\sec^2\theta\,d\theta$$

$x = \tan\theta$
$dx = \sec^2\theta\,d\theta$

$$= \int_0^{\pi/2} \cos\theta\,d\theta = \sin\theta\Big]_0^{\pi/2} = \sin\left(\frac{\pi}{2}\right) - \sin(0) = 1$$

$$W = 2\pi\int_0^\infty (\text{radius})\cdot(\text{height})\,dx = 2\pi\int_0^\infty x\cdot(x^2+1)^{-3/2}\,dx$$

$$= 2\pi\int_0^{\pi/2} \tan\theta\cdot(\sec^2\theta)^{-3/2}\sec^2\theta\,d\theta = 2\pi\int_0^{\pi/2}\sin\theta\,d\theta = 2\pi\cdot\Big[-\cos\theta\Big]_0^{\pi/2}$$

$x = \tan\theta$
$dx = \sec^2\theta\,d\theta$

$$= 2\pi\left(-\cos\left(\frac{\pi}{2}\right) + \cos(0)\right) = 2\pi$$

$$x_c = \frac{W}{2\pi A} = \frac{2\pi}{2\pi\cdot 1} = 1$$