

3. Evaluate the following integrals:

$$\text{a. } \int_0^\pi \sin x \cos x \sin(\pi \cos x) dx = -\frac{1}{\pi^2} \int_{\pi}^{-\pi} t \sin t dt = \frac{1}{\pi^2} \int_{-\pi}^{\pi} t \sin t dt$$

$$\begin{aligned} t &= \pi \cos x \\ dt &= -\pi \sin x dx \end{aligned}$$

$$= \frac{1}{\pi^2} \left(\left[-t \cos t \right]_{-\pi}^{\pi} + \int_{-\pi}^{\pi} \cos t dt \right) = \frac{1}{\pi^2} \left(-\pi \cos \pi - \pi \cos(-\pi) + \left[\sin t \right]_{-\pi}^{\pi} \right)$$

$$\begin{aligned} u = t &\Rightarrow du = dt \\ dv = \sin t dt &\Rightarrow v = -\cos t \end{aligned}$$

$$= \frac{1}{\pi^2} (\pi + \pi + \sin(\pi) - \sin(-\pi)) = \frac{2}{\pi}$$

$$\text{b. } \int \frac{\tan \theta}{(1 - \tan^2 \theta)^2} d\theta = \int \frac{\sin \theta \cos^3 \theta}{(\cos^2 \theta - \sin^2 \theta)^2} d\theta = \int \frac{\frac{1}{2} \sin 2\theta \cdot \cos^2 \theta}{\cos^2 2\theta} d\theta$$

$$= \frac{1}{4} \int \frac{\sin 2\theta \cdot (1 + \cos 2\theta)}{\cos^2 2\theta} d\theta = -\frac{1}{8} \int \left(\frac{1}{u^2} + \frac{1}{u} \right) du$$

$$\begin{aligned} u &= \cos 2\theta \\ du &= -2 \sin 2\theta d\theta \end{aligned}$$

$$= \frac{1}{8} \left(\frac{1}{u} - \ln|u| \right) + C = \frac{1}{8 \cos 2\theta} - \frac{1}{8} \ln|\cos 2\theta| + C$$