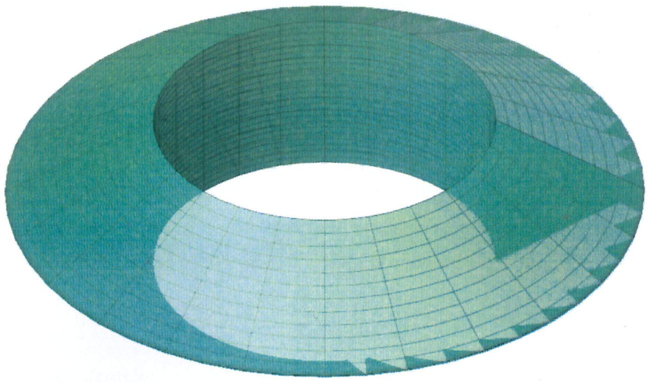
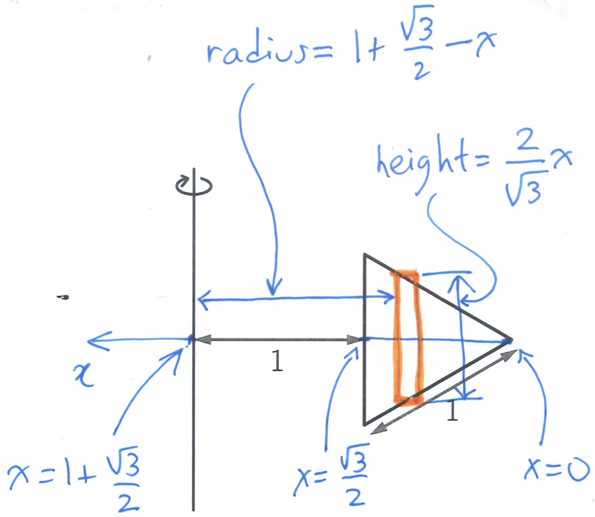
4a. A solid is generated by revolving an equilateral triangle with unit side length about a line at a unit distance from one of its sides as shown in the figure. Express the volume V of the solid as an integral using either ☐ the **washer** method or ☒ the **cylindrical shells** method by carefully defining your variable of integration, drawing a typical rectangle that generates a washer or a cylindrical shell and showing the relevant lengths and distances on the figure. [Indicate your method by ✗ing the corresponding ☐. Do not evaluate the integral!]

$$V = 2\pi \int_0^{\sqrt{3}/2} \underbrace{\left(1+\frac{\sqrt{3}}{2}-x\right)}_{\text{radius}} \cdot \underbrace{\frac{2}{\sqrt{3}}x}_{\text{height}}\, dx$$





4b. We start a rabbit farm with a pair of rabbits. Assume that at any moment the rabbit population is increasing at a rate proportional to the square of the rabbit population at that moment. Show that we will have infinitely many rabbits after a finite period of time.

Let N be the number of rabbits.

Then $\frac{dN}{dt} = k \cdot N^2$ for some positive constant k.

$\Downarrow$

$$\frac{dN}{N^2} = k\,dt \Rightarrow \int \frac{dN}{N^2} = \int k\,dt \Rightarrow -\frac{1}{N} = kt + C$$

Since $N(0)=2$, we have $-\frac{1}{2} = -\frac{1}{N(0)} = 0 + C \Rightarrow C = -\frac{1}{2}$

Hence $-\frac{1}{N} = kt - \frac{1}{2} \Rightarrow N = \frac{2}{1-2kt} \Rightarrow \lim_{t\to\left(\frac{1}{2k}\right)^-} N = \infty$

We will have infinitely many rabbits after a time of $\frac{1}{2k}$.