

3. Evaluate the following integrals.

$$\begin{aligned} \text{a. } \int_0^1 x^{4035} (x^4 + 1)^{2017} (3x^4 + 1) dx &= \int_0^1 (x^2)^{2017} \cdot (x^4 + 1)^{2017} \cdot x \cdot (3x^4 + 1) dx \\ &= \int_0^1 (x^6 + x^2)^{2017} \cdot (3x^5 + x) dx = \int_0^2 u^{2017} \cdot \frac{1}{2} du = \frac{1}{2} \cdot \frac{u^{2018}}{2018} \Big|_0^2 = \frac{2^{2018}}{2 \cdot 2018} = \frac{2^{2016}}{1009} \end{aligned}$$

$$\begin{aligned} u &= x^6 + x^2 \\ du &= (6x^5 + 2x) dx \end{aligned}$$

$$\begin{aligned} \text{b. } \int \frac{\sin x - \cos x}{1 + \sin 2x} dx &= \int \frac{\sin x - \cos x}{\sin^2 x + \cos^2 x + 2\sin x \cos x} dx \\ &= \int \frac{\sin x - \cos x}{(\sin x + \cos x)^2} dx = - \int \frac{du}{u^2} = \frac{1}{u} + C = \frac{1}{\sin x + \cos x} + C \end{aligned}$$

$$\begin{aligned} u &= \sin x + \cos x \\ du &= (\cos x - \sin x) dx \end{aligned}$$