2. Find the largest and smallest possible values of the area of the triangle cut off from the first quadrant by a line L which is tangent to the parabola $y = 15 - 2x - x^2$ at a point in the first quadrant.

[The *first quadrant* consists of the points (x, y) with $x \geq 0$ and $y \geq 0$.]

$y=0 \Rightarrow 15-2x-x^2=0 \Rightarrow x=3,\ x=-5$

Let a be the x-coordinate of the point where L is tangent to the parabola. Then $0 \leq a \leq 3$.

$y' = -2-2x \Rightarrow y'|_{x=a} = -2\cdot(a+1)$

Therefore the equation of L is: $y-(15-2a-a^2) = -2\cdot(a+1)\cdot(x-a)$

$y=0 \Rightarrow x = \dfrac{15-2a-a^2}{2\cdot(a+1)} + a = \dfrac{a^2+15}{2\cdot(a+1)}$

$x=0 \Rightarrow y = 15-2a-a^2+2\cdot(a+1)a = a^2+15$

The area of the triangle is $A = \dfrac{1}{2}\cdot\dfrac{a^2+15}{2\cdot(a+1)}\cdot(a^2+15)$.

We want to maximize/minimize $A = \dfrac{1}{4}\cdot\dfrac{(a^2+15)^2}{a+1}$ for $0 \leq a \leq 3$.

Critical points: $\dfrac{dA}{da} = \dfrac{1}{4}\cdot\left(\dfrac{2\cdot(a^2+15)\cdot 2a}{a+1} - \dfrac{(a^2+15)^2}{(a+1)^2}\right) = \dfrac{(a^2+15)\cdot(3a^2+4a-15)}{4\cdot(a+1)^2}$

$\dfrac{dA}{da} = 0 \Rightarrow 3a^2+4a-15=0 \Rightarrow a = \dfrac{5}{3},\ \cancel{a=-3}$ not in interval

$\Downarrow$

$A = \dfrac{800}{27}$

Endpoints:

$a=0 \Rightarrow A = \dfrac{225}{4}$

$a=3 \Rightarrow A = 36$

As $225 > 144 = 4\cdot 36$ and $27\cdot 36 = 972 > 800$, $\dfrac{225}{4}$ is the largest and $\dfrac{800}{27}$ is the smallest possible value of the area of the triangle.