2. Suppose that f is a differentiable function with $f(1) = 1/2$, $f'(1) = 3$, $f(2) = 7$, $f'(2) = -9$, $f(5) = 1/2$, $f'(5) = -4$.

a. Find an equation for the tangent line to the graph of $y = f(x)$ at the point with $x = 2$.

$$y - 7 = -9 \cdot (x - 2)$$

b. Compute $f(1.1)$ approximately.

$$f(1.1) \approx f(1) + f'(1) \cdot (1.1 - 1) = \frac{1}{2} + 3 \cdot \frac{1}{10} = \frac{4}{5} = 0.8$$

c. Find $\left.\dfrac{du}{dt}\right|_{(t,u)=(2,1)}$ if u is a differentiable function of t satisfying the relation $f(tf(u)) = f(1+t^2u^2)$.

d/dt

$$f'(tf(u)) \cdot \left(1 \cdot f(u) + t f'(u) \frac{du}{dt}\right) = f'(1+t^2u^2) \cdot \left(2tu^2 + t^2 \cdot 2u \frac{du}{dt}\right)$$

$$\Downarrow \ (t,u) = (2,1)$$

$$f'(2 \cdot f(1)) \cdot \left(f(1) + 2f'(1) \frac{du}{dt}\right) = f'(5) \cdot \left(4 + 8 \frac{du}{dt}\right)$$

$$\Downarrow$$

$$3 \cdot \left(\frac{1}{2} + 6 \frac{du}{dt}\right) = -4 \cdot \left(4 + 8 \frac{du}{dt}\right)$$

$$\Downarrow$$

$$\frac{du}{dt} = -\frac{7}{20} \quad \text{at} \quad (t,u) = (2,1)$$