

1. Evaluate the following limits without using L'Hôpital's Rule.

$$\begin{aligned}
 \text{a. } \lim_{x \rightarrow 2} \frac{x^3 - 3x^2 + 4}{\sqrt{x^3 + 8x} - \sqrt{5x^2 + 4}} &= \lim_{x \rightarrow 2} \left( \frac{x^3 - 3x^2 + 4}{(x^3 + 8x) - (5x^2 + 4)} \cdot (\sqrt{x^3 + 8x} + \sqrt{5x^2 + 4}) \right) \\
 &= \lim_{x \rightarrow 2} \frac{x^3 - 3x^2 + 4}{x^3 - 5x^2 + 8x - 4} \cdot \lim_{x \rightarrow 2} (\sqrt{x^3 + 8x} + \sqrt{5x^2 + 4}) \\
 &= \lim_{x \rightarrow 2} \frac{(x-2)^2 \cdot (x+1)}{(x-2)^2 \cdot (x-1)} \cdot (\sqrt{24} + \sqrt{24}) \\
 &= \lim_{x \rightarrow 2} \frac{x+1}{x-1} \cdot 4\sqrt{6} \\
 &= 3 \cdot 4\sqrt{6} = 12\sqrt{6}
 \end{aligned}$$

$$\text{b. } \lim_{x \rightarrow 0} \frac{\cos 8x - \cos 5x \cos 3x}{x^2} = \lim_{x \rightarrow 0} \frac{\cos(5x+3x) - \cos 5x \cdot \cos 3x}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{\cos 5x \cdot \cos 3x - \sin 5x \cdot \sin 3x - \cos 5x \cdot \cos 3x}{x^2}$$

$$= \lim_{x \rightarrow 0} \left( -5 \cdot 3 \cdot \frac{\sin 5x}{5x} \cdot \frac{\sin 3x}{3x} \right) = -5 \cdot 3 \cdot 1 \cdot 1 = -15$$