2. Evaluate the following.

a. $\displaystyle\int \frac{\tan x}{1+\sec 2x}\,dx = \int \frac{1}{1+\frac{1}{\cos 2x}}\cdot \tan x\,dx = \int \frac{\cos 2x}{1+\cos 2x}\cdot \tan x\,dx$

$\displaystyle = \int \frac{2\cos^2 x - 1}{2\cos^2 x}\cdot\frac{\sin x}{\cos x}\,dx = \int\left(\frac{1}{\cos x}-\frac{1}{2\cos^3 x}\right)\cdot \sin x\,dx$

$\displaystyle = \int\left(\frac{1}{u}-\frac{1}{2u^3}\right)\cdot(-du) = -\ln|u| - \frac{1}{4u^2} + C = -\ln|\cos x| - \frac{1}{4\cos^2 x} + C$

$\boxed{\begin{array}{l} u=\cos x \\ du=-\sin x\,dx \end{array}}$

b. $\displaystyle\int_1^2 \ln(x^2+x)\,dx = \int_1^2 \ln((x+1)\cdot x)\,dx = \int_1^2 (\ln(x+1)+\ln x)\,dx$

$= \Big[(x+1)\ln(x+1) - (x+1) + x\ln x - x\Big]_1^2$

$= 3\ln 3 - 3 + 2\ln 2 - 2 - 2\ln 2 + 2 - 1\cdot\underbrace{\ln 1}_{0} + 1$

$= 3\ln 3 - 2$

c. $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n}\frac{in^2}{n^4+i^4} = \lim_{n\to\infty}\underbrace{\sum_{i=1}^{n}\frac{\frac{i}{n}}{1+\left(\frac{i}{n}\right)^4}\cdot\frac{1}{n}}_{\circledast} = \int_0^1 \frac{x}{1+x^4}\,dx = \frac{1}{2}\int_0^1 \frac{du}{1+u^2}$

$\boxed{\begin{array}{l} u=x^2 \\ du=2x\,dx \end{array}}$

$\displaystyle = \frac{1}{2}\arctan u\Big]_0^1 = \frac{1}{2}\underbrace{\arctan 1}_{\frac{\pi}{4}} - \frac{1}{2}\underbrace{\arctan 0}_{0} = \frac{\pi}{8}$

$\circledast$ = a right Riemann sum for $f(x) = \dfrac{x}{1+x^4}$ on $[0,1]$