

4. Consider the region R between the graph of $y = x \sin(x^3)$ and the x -axis for $0 \leq x \leq \pi^{1/3}$.

a. Find the volume V of the solid generated by revolving R about the x -axis.

$$\begin{aligned}
 V &= \pi \int_0^{\pi^{1/3}} (x \sin(x^3))^2 dx \\
 &= \pi \int_0^{\pi^{1/3}} x^2 \sin^2(x^3) dx \\
 &= \pi \int_0^{\pi^{1/3}} x^2 \cdot \frac{1 - \cos(2x^3)}{2} dx \\
 &= \frac{\pi}{2} \int_0^{\pi^{1/3}} x^2 dx - \frac{\pi}{2} \int_0^{\pi^{1/3}} x^2 \cos(2x^3) dx \\
 &= \frac{\pi}{2} \cdot \left[\frac{1}{3} x^3 \right]_0^{\pi^{1/3}} - \frac{\pi}{2} \int_0^{2\pi} \cos u \cdot \frac{1}{6} du \quad \boxed{u = 2x^3} \\
 &= \frac{\pi^2}{6} - \frac{\pi}{12} [\sin u]_0^{2\pi} = \frac{\pi^2}{6}
 \end{aligned}$$

b. Find the volume W of the solid generated by revolving R about the y -axis.

$$\begin{aligned}
 W &= 2\pi \cdot \int_0^{\pi^{1/3}} x \cdot x \sin(x^3) dx \\
 &= 2\pi \int_0^{\pi^{1/3}} x^2 \sin(x^3) dx \quad \boxed{u = x^3} \\
 &= 2\pi \int_0^{\pi} \sin u \cdot \frac{1}{3} du \quad \boxed{du = 3x^2 dx} \\
 &= \frac{2\pi}{3} [-\cos u]_0^{\pi} = \frac{4\pi}{3}
 \end{aligned}$$