

3a. Determine $f(5)$ if f is a continuous function that satisfies $\int_0^{4x+\sin(\pi x)} f(t) dt = x^2$ for all x .

$$\frac{d}{dx} \int_0^{4x+\sin(\pi x)} f(t) dt = \frac{d}{dx} x^2$$

FTC 1

$$f(4x+\sin(\pi x)) \cdot (4+\pi \cos(\pi x)) = 2x$$

$$x = \frac{3}{2}$$

$$f\left(4 \cdot \frac{3}{2} + \sin\left(\frac{3\pi}{2}\right)\right) \cdot \left(4 + \pi \cos\left(\frac{3\pi}{2}\right)\right) = 2 \cdot \frac{3}{2}$$

$$f(5) = \frac{3}{4}$$

$$(4x + \sin \pi x)' = 4 + \pi \cos \pi x \geq 4 - \pi > 0 \Rightarrow 4x + \sin \pi x \text{ is increasing}$$

$$\text{On } (-\infty, \infty) \Rightarrow 4x + \sin(\pi x) = 5 \text{ only when } x = \frac{3}{2}$$

3b. Determine $f(5)$ if f is a function that satisfies $\int_0^{f(x)} t^2 dt = 4x + \sin(\pi x)$ for all x .

$$\left[\frac{1}{3} t^3 \right]_0^{f(x)} = 4x + \sin(\pi x)$$

$$\frac{1}{3} f(x)^3 = 4x + \sin(\pi x)$$

$$f(x) = (12x + 3 \sin(\pi x))^{1/3}$$

$$x = 5$$

$$f(5) = (12 \cdot 5 + 3 \sin 5\pi)^{1/3}$$

$$f(5) = 60^{1/3}$$