

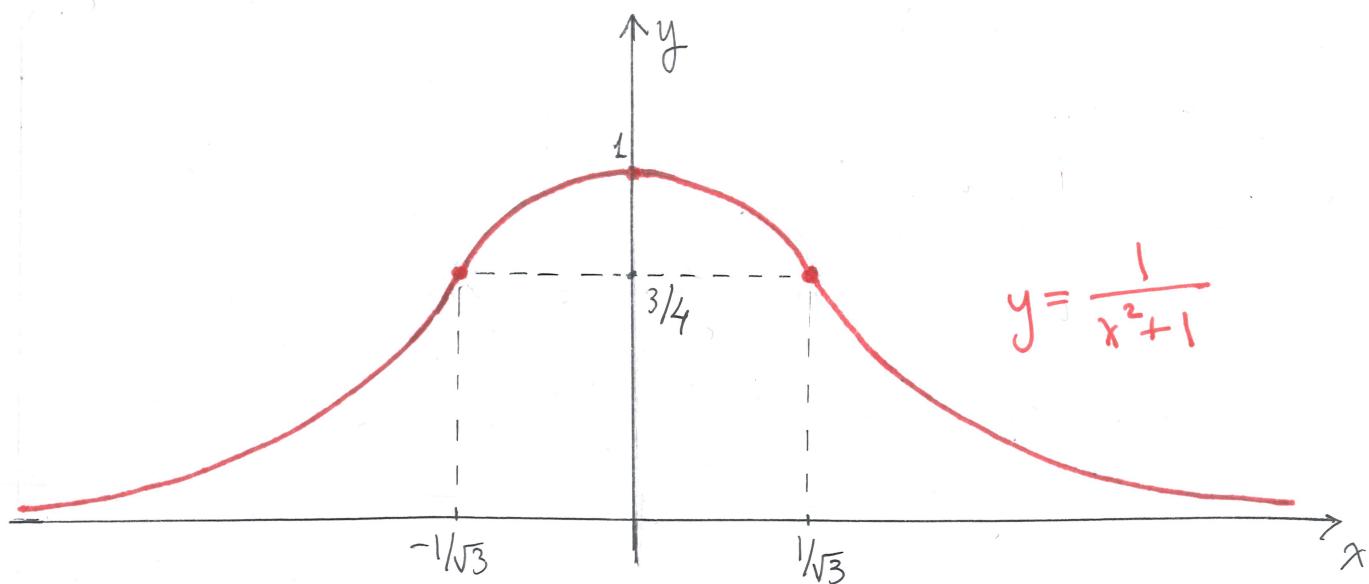
- 1a. Sketch the graph of $y = 1/(x^2 + 1)$ by computing y' and y'' , and determining their signs; finding the critical points, the inflection points, the intercepts, and the asymptotes; and clearly labeling them in the picture.

$$y' = -\frac{2x}{(x^2+1)^2} = 0 \Rightarrow x=0 \Rightarrow y=1$$

$$y'' = -2 \cdot \frac{1 \cdot (x^2+1)^2 - x \cdot 2(x^2+1) \cdot 2x}{(x^2+1)^4} = \frac{2 \cdot (3x^2-1)}{(x^2+1)^3} = 0 \Rightarrow x = \pm \frac{1}{\sqrt{3}} \Rightarrow y = \frac{3}{4}$$

x	$-1/\sqrt{3}$	0	$1/\sqrt{3}$			
y'	+	+	0	-	-	
y''	+	0	-	-	0	+
	↑ inf pt	loc max	↑ inf pt			

$$\lim_{x \rightarrow \infty \text{ or } -\infty} y = 0$$



- 1b. Show that $\left| \frac{1}{a^2+1} - \frac{1}{b^2+1} \right| \leq \frac{3\sqrt{3}}{8} |a-b|$ for all real numbers a, b .

Since $f(x) = 1/(x^2+1)$ is differentiable hence continuous everywhere, by MVT
 $f'(c) = \frac{f(a)-f(b)}{a-b}$ for some c between a and b . By Parta, critical points
of f' are $x = \pm \frac{1}{\sqrt{3}}$, and $f'(\pm \frac{1}{\sqrt{3}}) = \mp \frac{3\sqrt{3}}{8}$. Moreover, $\lim_{x \rightarrow \infty \text{ or } -\infty} f'(x) = 0$.

Hence $|f'(x)| \leq \frac{3\sqrt{3}}{8}$ for all x . In particular, $|f'(c)| \leq \frac{3\sqrt{3}}{8}$ and
therefore $\left| \frac{1}{a^2+1} - \frac{1}{b^2+1} \right| \leq \frac{3\sqrt{3}}{8} \cdot |a-b|$.