4. Air is escaping from a balloon that is hanging at the end of a 75 cm long string attached to a nail on a wall. Assume that at all times the balloon has the shape of a sphere which is tangent to the wall, and the string extends along a line which passes through the center of the sphere and lies in a vertical plane perpendicular to the wall.

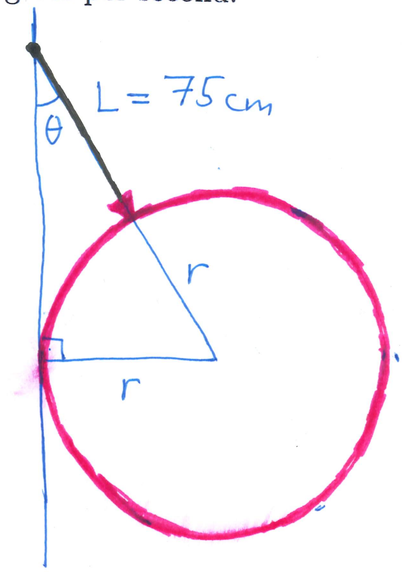
Suppose that at a certain moment the volume of the balloon is decreasing at a rate of 1800π cm^3/s and its surface area is decreasing at a rate of 240π cm^2/s. Determine how fast the angle between the string and the wall is changing at this moment. Express your answer in units of degrees per second.

r = radius of the balloon

V = volume of the balloon

A = surface area of the balloon

θ = angle between the string and the wall



$$V = \frac{4\pi}{3} r^3 \quad \Downarrow d/dt \quad \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$A = 4\pi r^2 \quad \Downarrow d/dt \quad \frac{dA}{dt} = 8\pi r \frac{dr}{dt}$$

at our moment

$$-1800\pi = 4\pi r^2 \frac{dr}{dt} \qquad -240\pi = 8\pi r \frac{dr}{dt}$$

$$r = 15 \text{ cm}, \quad \frac{dr}{dt} = -2 \text{ cm/s}$$

$$\sin\theta = \frac{r}{L+r} \overset{d/dt}{\Longrightarrow} \cos\theta \cdot \frac{d\theta}{dt} = \frac{L}{(L+r)^2} \cdot \frac{dr}{dt}$$

at our moment

$$\Longrightarrow \frac{\sqrt{90^2-15^2}}{90} \frac{d\theta}{dt} = \frac{75}{90^2} \cdot (-2 \text{ cm/s}) \Longrightarrow \frac{d\theta}{dt} = -\frac{1}{9\sqrt{35}} \text{ radians/s}$$

$$\Longrightarrow \frac{d\theta}{dt} = -\frac{1}{9\sqrt{35}} \cdot \frac{180}{\pi} \, °/s = -\frac{4}{\pi}\sqrt{\frac{5}{7}} \, °/s$$

The angle is decreasing at a rate of $\frac{4}{\pi}\sqrt{\frac{5}{7}}$ °/s.