3. Suppose that f is a twice-differentiable function satisfying the following conditions:

- $y = x/2 + 1$ is an equation for the tangent line to the graph of $y = f(x)$ at the point with $x = 4$.
- $y = x/4 - 3$ is an equation for the tangent line to the graph of $y = f(x)$ at the point with $x = 8$.
- $y = -x/3 + 2$ is an equation for the tangent line to the graph of $y = f(x)$ at the point with $x = 12$.

Consider the function $g(x) = (f(x^3))^2$.

a. Find an equation for the tangent line to the graph of $y = g(x)$ at the point with $x = 2$.

$f'(8) = \frac{1}{4}$, $f(8) = \frac{8}{4} - 3 = -1$

$g(2) = (f(8))^2 = (-1)^2 = 1$

$g'(x) = 2f(x^3)\, f'(x^3) \cdot 3x^2$

$g'(2) = 2f(8)\, f'(8) \cdot 3 \cdot 2^2 = 2 \cdot (-1) \cdot \frac{1}{4} \cdot 12 = -6$

An equation for the tangent line is:

$y - 1 = -6 \cdot (x - 2) \Rightarrow y = -6x + 13$

b. Suppose that $g''(2) = 0$. Find $f''(8)$.

$g''(x) = 2f'(x^3)\, f'(x^3) \cdot (3x^2)^2 + 2f(x^3)\, f''(x^3) \cdot (3x^2)^2$
$+ 2f(x^3)\, f'(x^3) \cdot 6x$

$\Rightarrow g''(2) = 2f'(8)\, f'(8) \cdot (3 \cdot 2^2)^2 + 2f(8)\, f''(8) \cdot (3 \cdot 2^2)^2 + 2f(8)\, f'(8) \cdot 6 \cdot 2$

$\Rightarrow 0 = 2 \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot 12^2 + 2 \cdot (-1) \cdot f''(8) \cdot 12^2 + 2 \cdot (-1) \cdot \frac{1}{4} \cdot 12$

$\Rightarrow 288 f''(8) = 12 \Rightarrow f''(8) = \frac{1}{24}$