2. In each of the following, if the given statement is true for all functions f that are defined on the entire real line, then mark the ☐ to the left of TRUE with a ✗; otherwise, mark the ☐ to the left of FALSE with a ✗ and give a counterexample. No explanation is required.

a. If f is continuous on $(-\infty, \infty)$, then $f'(\pi)$ exists.

☐ TRUE

☒ FALSE, because it does not hold for $f(x) =$ $|x-\pi|$

b. If $f(-1) = -1$ and $f(1) = 1$, then $f(c) = 0$ for some c in $(-1, 1)$.

☐ TRUE

☒ FALSE, because it does not hold for $f(x) =$ $\begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$

c. If $f''(x) = -f(x)$ for all x, then $f(x) = \sin x$ or $f(x) = \cos x$.

☐ TRUE

☒ FALSE, because it does not hold for $f(x) =$ 0

d. If f is differentiable on $(-\infty, \infty)$, then $f'(\pi)$ exists.

☒ TRUE

☐ FALSE, because it does not hold for $f(x) =$

e. If $\lim_{x\to 0} |f(x)| = 1$, then $\lim_{x\to 0} f(x) = 1$ or $\lim_{x\to 0} f(x) = -1$.

☐ TRUE

☒ FALSE, because it does not hold for $f(x) =$ $\begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$