

1. Consider the functions

$$f(x) = \sqrt{x^2 + a} + bx \quad \text{and} \quad g(x) = x^3 + cx^2 + d$$

where a, b, c, d are nonzero real constants.

a. Give values to a, b, c, d by filling in the boxes below so that the resulting functions satisfy $f(1) = 0$ and $g(1) = 0$. No explanation is required in this part.

$$a = \boxed{3}$$

$$b = \boxed{-2}$$

$$c = \boxed{1}$$

$$d = \boxed{-2}$$

b. Compute the limit $\lim_{x \rightarrow 1} \frac{f(x)}{g(x)}$ where a, b, c, d have the values given in Part a.

[Do not use L'Hôpital's Rule. This part will be graded only if Part a is completely correct.]

$$\lim_{x \rightarrow 1} \frac{f(x)}{g(x)} = \lim_{x \rightarrow 1} \frac{\sqrt{x^2 + 3} - 2x}{x^3 + x^2 - 2} = \lim_{x \rightarrow 1} \frac{\sqrt{x^2 + 3} - 2x}{x^3 + x^2 - 2} \cdot \frac{\sqrt{x^2 + 3} + 2x}{\sqrt{x^2 + 3} + 2x}$$

$$= \lim_{x \rightarrow 1} \frac{x^2 + 3 - 4x^2}{(x-1)(x^2 + 2x + 2) \cdot (\sqrt{x^2 + 3} + 2x)}$$

$$= \lim_{x \rightarrow 1} \frac{-3 \cdot (x^2 - 1)}{(x-1) \cdot (x^2 + 2x + 2) \cdot (\sqrt{x^2 + 3} + 2x)}$$

$$= \lim_{x \rightarrow 1} \frac{-3 \cdot (x-1)(x+1)}{(x-1) \cdot (x^2 + 2x + 2) \cdot (\sqrt{x^2 + 3} + 2x)}$$

$$= \lim_{x \rightarrow 1} \frac{-3 \cdot (x+1)}{(x^2 + 2x + 2) \cdot (\sqrt{x^2 + 3} + 2x)}$$

$$= \frac{-3 \cdot 2}{5 \cdot (2+2)} = -\frac{3}{10}$$