

5. Evaluate the following integrals.

$$\text{a. } \int_0^1 x^5 \sqrt[4]{1-x^3} dx = -\frac{1}{3} \int_1^0 (1-u) \cdot u^{1/4} du = \frac{1}{3} \int_0^1 (u^{1/4} - u^{5/4}) du$$

$$\begin{aligned} u &= 1-x^3 \\ du &= -3x^2 dx \end{aligned}$$

$$= \frac{1}{3} \left[\frac{u^{5/4}}{5/4} - \frac{u^{9/4}}{9/4} \right]_0^1 = \frac{1}{3} \left(\frac{4}{5} - \frac{4}{9} \right) = \frac{16}{135}$$

$$\text{b. } \int \sin 2x \tan^2 x dx = \int 2 \sin x \cos x \cdot \frac{\sin^2 x}{\cos^2 x} dx = 2 \int \frac{1 - \cos^2 x}{\cos x} \cdot \sin x dx$$

$$= -2 \int \left(\frac{1}{u} - u \right) du = -2 \ln|u| + u^2 + C = \cos^2 x - 2 \ln|\cos x| + C$$

$$\begin{aligned} u &= \cos x \\ du &= -\sin x dx \end{aligned}$$