

2. In each of the following, indicate whether the given statement is TRUE or FALSE by marking the corresponding \square with a \times , and then explain why it is true or false.

a. $\int \sin^3 x dx = \frac{\sin^4 x}{4} + C$ TRUE FALSE

$$\left(\frac{\sin^4 x}{4}\right)' = \sin^3 x \cos x \neq \sin^3 x \quad \text{at } x = \frac{\pi}{2}$$

b. $\int \sin^3 x dx = \frac{\sin^4 x}{4 \cos x} + C$ TRUE FALSE

$$\left(\frac{\sin^4 x}{4 \cos x}\right)' = \sin^3 x + \frac{\sin^5 x}{4 \cos^2 x} \neq \sin^3 x \quad \text{at } x = \frac{\pi}{4}$$

c. $\int \sin^3 x dx = \frac{1}{6} \cos x \cos 2x - \frac{5}{6} \cos x + C$ TRUE FALSE

$$\begin{aligned} \left(\frac{1}{6} \cos x \cos 2x - \frac{5}{6} \cos x\right)' &= \left(\frac{1}{6} \cos x \cdot (2 \cos^2 x - 1) - \frac{5}{6} \cos x\right)' \\ &= \left(\frac{1}{3} \cos^3 x - \cos x\right)' = -\cos^2 x \sin x + \sin x = \sin^3 x \quad \text{for all } x \end{aligned}$$

d. $\int \frac{dx}{x^2+1} = \frac{\ln(x^2+1)}{2x} + C$ TRUE FALSE

$$\left(\frac{\ln(x^2+1)}{2x}\right)' = \frac{1}{x^2+1} - \frac{\ln(x^2+1)}{2x^2} \neq \frac{1}{x^2+1} \quad \text{at } x=1$$

e. $\int \frac{dx}{x^2+1} = \arcsin\left(\frac{x}{\sqrt{x^2+1}}\right) + C$ TRUE FALSE

$$\left(\arcsin\left(\frac{x}{\sqrt{x^2+1}}\right)\right)' = \frac{1}{\sqrt{1-\left(\frac{x}{\sqrt{x^2+1}}\right)^2}} \cdot \frac{\sqrt{x^2+1} - \frac{x^2}{\sqrt{x^2+1}}}{x^2+1} = \frac{1}{x^2+1} \quad \text{for all } x$$