

4. An island has the shape of a  $10 \text{ hm} \times 10 \text{ hm}$  square and its landscape consists of a mountain whose height  $h$  at a horizontal distance  $x$  from the shore is given by  $h = x^2$  where both  $h$  and  $x$  are measured in hectometers (=hm). Let  $V$  be the volume of the mountain.

a. Express  $V$  as an integral with respect to  $h$  by considering cutting the mountain into slices as shown in *Figure a*.

$$V = \int_0^{25} \underbrace{(10 - 2\sqrt{h})^2}_{\text{area of the cross-section}} dh$$

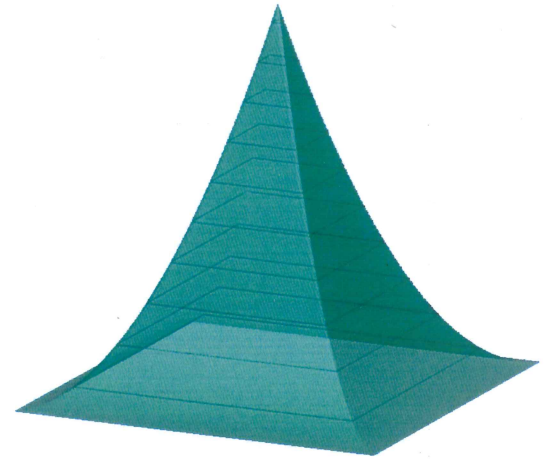


Figure a:

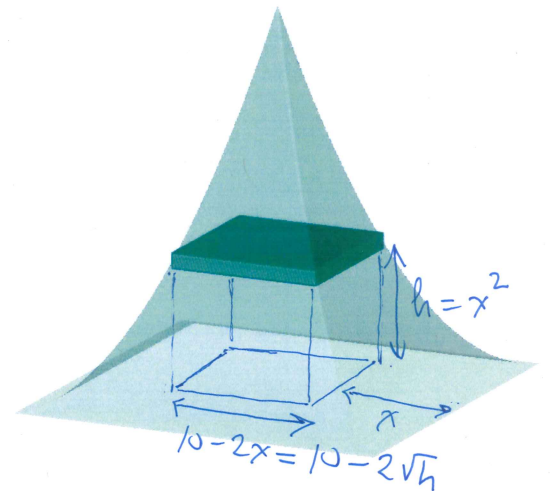
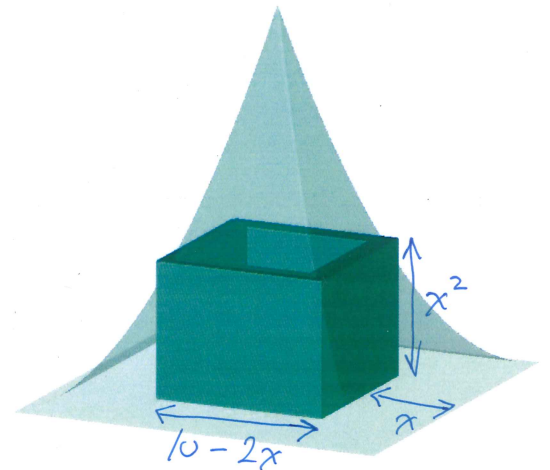


Figure b:

$$V = \int_0^5 \underbrace{4 \cdot (10 - 2x)}_{\text{circumference of the shell}} \cdot \underbrace{x^2}_{\text{height of the shell}} dx$$



c. Compute  $V$ .

$$V = 8 \int_0^5 (5x^2 - x^3) dx = 8 \cdot \left[ \frac{5}{3} x^3 - \frac{1}{4} x^4 \right]_0^5 = 8 \cdot 5^4 \cdot \left( \frac{1}{3} - \frac{1}{4} \right) = \frac{1250}{3}$$