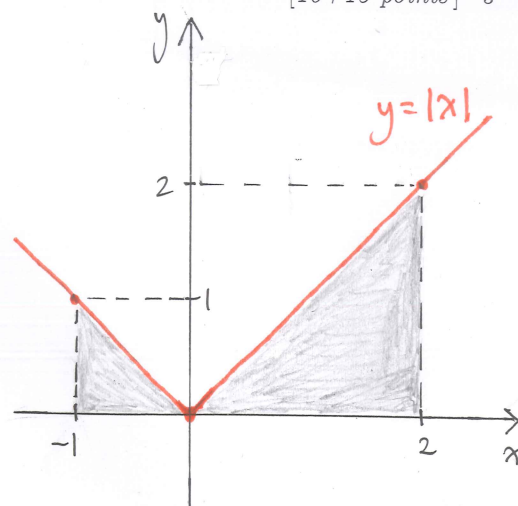


3a. Find the average value of $|x|$ on the interval $[-1, 2]$.

$$\text{Average} = \frac{\int_{-1}^2 |x| dx}{2 - (-1)} = \frac{\frac{1}{2} \cdot 1 \cdot 1 + \frac{1}{2} \cdot 2 \cdot 2}{3} = \frac{5}{6}$$



3b. Suppose that a continuous function f satisfies the equation

$$f(x) = x^2 - x + (1-x) \int_0^x t^2 f(t) dt + x \int_x^1 (t-t^2) f(t) dt$$

for all x . Express $f''(1/2)$ in terms of $A = f(1/2)$.

$$f'(x) = 2x - 1 - \int_0^x t^2 f(t) dt + (1-x)x^2 f(x) + \int_x^1 (t-t^2) f(t) dt - x \cdot (x-x^2) f(x)$$

$$f'(x) = 2x - 1 - \int_0^x t^2 f(t) dt + \int_x^1 (t-t^2) f(t) dt$$

$$f''(x) = 2 - x^2 f(x) - (x-x^2) f(x)$$

$$f''(x) = 2 - x f(x) \Rightarrow f''(1/2) = 2 - \frac{1}{2} f(1/2) = 2 - \frac{1}{2} A$$