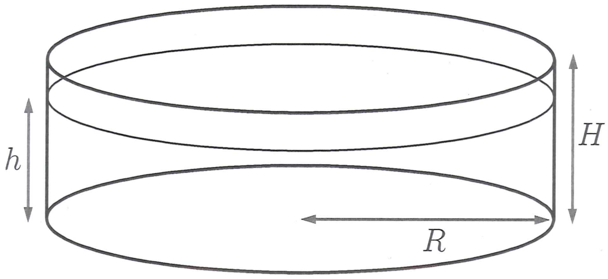
2. A pool, like the one in front of the Faculty of Science Building A, loses water from its sides and its bottom due to seepage, and from its top due to evaporation. For a pool with radius R and depth H in meters, the rate of this loss in m^3/hour is given by an expression of the form

$$aR^2 + bR^2h + cRh^2$$

where h is the depth of the water in meters, and a, b, c are constants independent of R, H and h. Due to this loss, water must be pumped into the pool to keep it at the same level even when the drains are closed.



Suppose that $a = 1/300$ m/hour and $b = c = 1/150$ 1/hour. Find the dimensions of the pool with a volume of 45π m^3 which will require the water to be pumped at the slowest rate to keep it completely full.

$$\pi R^2 H = 45\pi \Rightarrow H = \frac{45}{R^2}$$

When the pool is full, the water is lost at the rate

$$L = aR^2 + bR^2H + cRH^2 = \frac{1}{300}R^2 + \frac{1}{150}R^2 \cdot \frac{45}{R^2} + \frac{1}{150}R\left(\frac{45}{R^2}\right)^2.$$

Hence we want to:

Minimize $L = \frac{1}{300}R^2 + \frac{3}{10} + \frac{27}{2}\cdot\frac{1}{R^3}$ for $0 < R < \infty$

Critical points: $\frac{dL}{dR} = \frac{1}{150}R - \frac{81}{2}\cdot\frac{1}{R^4} = 0 \Rightarrow R = 3\cdot 5^{2/5}$ m

"Endpoints": $R = 0$ m : $\lim_{R\to 0^+} L = \infty$

$R = \infty$ m : $\lim_{R\to\infty} L = \infty$

$R = 3\cdot 5^{2/5}$ m gives the slowest rate.

$$R = 3\cdot 5^{2/5}\text{ m} \Rightarrow H = \frac{45}{(3\cdot 5^{2/5})^2}\text{ m} = 5^{1/5}\text{ m}.$$

The pool that requires the water to be pumped at the slowest rate has $R = 3\cdot 5^{2/5}$ m and $H = 5^{1/5}$ m.