1. Suppose you are told that a function f satisfies the condition:

$$x^3 - 3x - 3 \le f(x) \le 9x^2 - 27x + 17 \qquad \text{for} \quad |x-2| < 1 \qquad (*)$$

If YES, show how; if NO, explain why.

a. Is it possible to determine the value of $\lim_{x\to 2} f(x)$ using only this information? ☑ YES ☐ NO

$$\lim_{x\to 2}(x^3-3x-3) = 2^3 - 3\cdot 2 - 3 = -1$$

$$\|$$

$$\lim_{x\to 2}(9x^2-27x+17) = 9\cdot 2^2 - 27\cdot 2 + 17 = -1$$

$\Rightarrow$ By Squeeze Theorem, $\lim_{x\to 2} f(x) = -1$

b. At this point you peek at your neighbor's paper, and see that YES is marked in **Part 1c** and then the solution starts with the sentence:

Taking derivatives of all sides, we obtain $(x^3-3x-3)' \le f'(x) \le (9x^2-27x+17)'$ *for* $|x-2|<1$.

Just fill in the boxes.

You immediately realize that this cannot be true, because $\frac{d}{dx}(x^3-3x-3) = \boxed{3x^2-3}$ and $\frac{d}{dx}(9x^2-27x+17) = \boxed{18x-27}$, and if we take $a = \boxed{3/2}$, then $|a-2|<1$ but

$$\left.\frac{d}{dx}(x^3-3x-3)\right|_{x=a} = \boxed{15/4} > \boxed{0} = \left.\frac{d}{dx}(9x^2-27x+17)\right|_{x=a}.$$

If Yes, show how; if No, explain why.

c. Is it possible to determine the value of $f'(2)$ using only this information? ☑ Yes ☐ No

$(*) \Rightarrow \quad x=2 \Rightarrow \quad -1 \le f(2) \le -1 \quad \Rightarrow f(2) = -1$

$$(x+1)^2\cdot(x-2) = x^3-3x-2 \le f(x)-f(2) \le 9x^2-27x+18 = 9(x-1)(x-2)$$

$x<2$ $\qquad\qquad$ $x>2$

$$(x+1)^2 \ge \frac{f(x)-f(2)}{x-2} \ge 9(x-1) \qquad\qquad (x+1)^2 \le \frac{f(x)-f(2)}{x-2} \le 9(x-1)$$

Squeeze Theorem $\qquad \boxed{\lim_{x\to 2}(x+1)^2 = 9 = \lim_{x\to 2} 9(x-1)} \qquad$ Squeeze Theorem

$$\lim_{x\to 2^-}\frac{f(x)-f(2)}{x-2} = 9 \qquad\qquad \lim_{x\to 2^+}\frac{f(x)-f(2)}{x-2} = 9$$

$$f'(2) = \lim_{x\to 2}\frac{f(x)-f(2)}{x-2} = 9$$