

4. Evaluate the limit $\lim_{x \rightarrow \infty} \left(x \left(x \left(x \left(e^{1/x} - 1 \right) - 1 \right) - \frac{1}{2} \right) \right)$.

$$\lim_{x \rightarrow \infty} \left(x \left(x \left(x \left(e^{1/x} - 1 \right) - 1 \right) - \frac{1}{2} \right) \right)$$

$$= \lim_{x \rightarrow \infty} \frac{e^{1/x} - 1 - \frac{1}{x} - \frac{1}{2} \cdot \frac{1}{x^2}}{\frac{1}{x^3}}$$

$$\boxed{x = \frac{1}{t}} \quad \lim_{t \rightarrow 0^+} \frac{e^t - 1 - t - \frac{1}{2}t^2}{t^3} \rightarrow \frac{0}{0}$$

$$\stackrel{\text{L'H}}{\downarrow} \lim_{t \rightarrow 0^+} \frac{e^t - 1 - t}{3t^2} \rightarrow \frac{0}{0}$$

$$\stackrel{\text{L'H}}{\downarrow} \lim_{t \rightarrow 0^+} \frac{e^t - 1}{6t} \rightarrow \frac{0}{0}$$

$$\stackrel{\text{L'H}}{\downarrow} \lim_{t \rightarrow 0^+} \frac{e^t}{6} = \frac{e^0}{6} = \frac{1}{6}$$