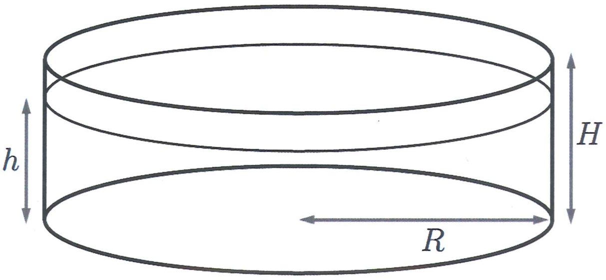
3. A pool, like the one in front of the Faculty of Science Building A, loses water from its sides and its bottom due to seepage. For a pool with radius R and depth H in meters, the rate of this loss in m^3/hour is given by

$$\frac{dV}{dt} = -aR^2h - bRh^2$$

where V is the volume of the water in cubic meters, t is the time in hours, h is the depth of the water in meters, and a and b are constants independent of R, H and h.



Consider a pool with $H = 1$ m, $R = 6$ m and $a = b = \pi/500$ 1/hour.

a. Find the depth of the water h as a function of time t if $h = 1$ m when $t = 0$ hour.

$$V = \pi R^2 h \Rightarrow \pi R^2 \frac{dh}{dt} = -\frac{\pi}{500}R^2 h - \frac{\pi}{500} R h^2$$

$$\Rightarrow 6^2 \cdot \frac{dh}{dt} = -\frac{1}{500} \cdot 6^2 \cdot h - \frac{1}{500} \cdot 6 \cdot h^2 \Rightarrow \frac{6\,dh}{h \cdot (h+6)} = -\frac{dt}{500}$$

$$\Rightarrow \left(\frac{1}{h} - \frac{1}{h+6}\right) dh = -\frac{dt}{500} \Rightarrow \ln h - \ln(h+6) = -\frac{t}{500} + C$$

$$\Rightarrow \frac{h}{h+6} = A e^{-t/500} \Rightarrow h = \frac{6}{7e^{t/500} - 1}$$

$t=0,\ h=1$

$$\boxed{A = \frac{1}{7}}$$

b. The term aR^2h on the right side of the equation represents the rate of loss due to seepage through the bottom of the pool. Find the total volume of the water that seeps through the bottom while the initially full pool completely empties.

$$\text{Volume} = \int_{t=0}^{t=\infty} aR^2 h\, dt = \frac{\pi}{500} \cdot 6^2 \cdot \int_0^\infty \frac{6}{7e^{t/500} - 1}\, dt$$

$$= 216\pi \int_0^\infty \frac{e^{-t/500}\, dt/500}{7 - e^{-t/500}} = 216\pi \int_6^7 \frac{du}{u} = 216\pi \ln\left(\frac{7}{6}\right) \text{ m}^3$$

$$\boxed{\begin{aligned} u &= 7 - e^{-t/500} \\ du &= e^{-t/500} \frac{dt}{500} \end{aligned}}$$