

4. Evaluate the following integrals.

a.  $\int_0^1 \frac{dx}{(1+\sqrt{x})^4} = \int_1^2 \frac{2(u-1)}{u^4} du = 2 \int_1^2 (u^{-3} - u^{-4}) du = 2 \left[ \frac{u^{-2}}{-2} - \frac{u^{-3}}{-3} \right]_1^2$

$u = 1 + \sqrt{x}$   
 $du = \frac{dx}{2\sqrt{x}} \Rightarrow dx = 2(u-1) du$

$$= 2 \left( -\frac{1}{8} + \frac{1}{2} + \frac{1}{24} - \frac{1}{3} \right)$$

$$= \frac{1}{6}$$

b.  $\int x^3 \sqrt{x^2+1} dx = \int (u-1) \cdot u^{1/2} \cdot \frac{1}{2} du = \frac{1}{2} \int (u^{3/2} - u^{1/2}) du$

$u = x^2 + 1$   
 $du = 2x dx$

$$= \frac{1}{2} \left( \frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2} \right) + C$$

$$= \frac{1}{5} (x^2+1)^{5/2} - \frac{1}{3} (x^2+1)^{3/2} + C$$