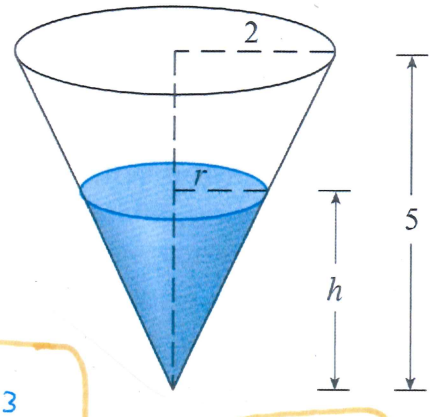
3. A water tank has the shape of an upside-down cone with radius 2 m and height 5 m. The water is running out of the tank through a small hole at the bottom. Assume that the speed of the water flowing through the hole is proportional to the square root of the depth of the water in the tank. ❁

If it takes 3 minutes for the depth of the water to decrease from 5 m to 4 m, find how long it takes for the full tank to completely drain.



V = Volume of water $= \frac{\pi}{3} r^2 h = \frac{\pi}{3}\left(\frac{2}{5}h\right)^2 h = \frac{4\pi}{75} h^3$

$\frac{r}{h} = \frac{2}{5}$

$\Downarrow d/dt$

$\frac{dV}{dt} = \frac{4\pi}{25} h^2 \frac{dh}{dt}$

❁ $\Rightarrow \frac{dV}{dt} = -k\sqrt{h}$ for some positive constant k

$\Downarrow$

$\frac{4\pi}{25} h^2 \frac{dh}{dt} = -k\sqrt{h} \Rightarrow \frac{4\pi}{25} h^{3/2}\, dh = -k\, dt$

$\Downarrow S$

$\frac{8\pi}{125} h^{5/2} = -kt + C$

$h^{5/2} = -at + b$ for some constants a, b.

$t = 0$ min, $h = 5$ m

$5^{5/2} = b$

$\Downarrow$

$h^{5/2} = -at + 5^{5/2}$

$t = 3$ min, $h = 4$ m

$4^{5/2} = -a \cdot 3 + 5^{5/2}$

$h = 0$ m

$t = \frac{5^{5/2}}{a} = \frac{3 \cdot 5^{5/2}}{5^{5/2} - 4^{5/2}}$ min

The full tank drains in $\frac{3}{1 - (4/5)^{5/2}}$ minutes.