3. Consider the curve defined by the equation $x^4 + y^4 = 4xy + 52$.

a. Find all points on the curve at which the tangent line is horizontal.

d/dx

$$4x^3+4y^3y'=4y+4xy'$$

$$\Downarrow \; \boxed{y'=0}$$

$$y=x^3$$

$$x^4+x^{12}=4x^4+52$$

$$\Downarrow$$

$$x^{12}-3x^4-52=0$$

$$\Downarrow$$

$$x=\sqrt{2} \text{ or } x=-\sqrt{2} \Leftarrow x^4=4 \Leftarrow (x^4-4)\cdot\underbrace{(x^8+4x^4+13)}_{\text{has no real roots as } 4^2-4\cdot 13=-36<0}=0$$

$$\Downarrow \qquad \Downarrow$$

$$y=2\sqrt{2} \qquad y=-2\sqrt{2}$$

$$\Rightarrow (x,y)=(\sqrt{2},2\sqrt{2}),\ (-\sqrt{2},-2\sqrt{2})$$

b. Find $\dfrac{d^2y}{dx^2}$ at all points where the tangent line is horizontal.

d/dx

$$3x^2+3y^2(y')^2+y^3y''=y'+y'+xy''$$

$$\Downarrow \; \boxed{y'=0}$$

$$3x^2+y^3y''=xy''$$

$$\Downarrow$$

$$6\pm16\sqrt{2}y''=\pm\sqrt{2}y'' \Rightarrow y''=\mp\frac{\sqrt{2}}{5}$$

$$\Rightarrow y''=-\frac{\sqrt{2}}{5} \text{ at } (\sqrt{2},2\sqrt{2}) \text{ and } \frac{\sqrt{2}}{5} \text{ at } (-\sqrt{2},-2\sqrt{2})$$