

2. Consider the function  $f(x) = \sin\left(\frac{\pi}{\sin\left(\frac{\pi}{x}\right)}\right)$ .

a. Find an equation of the tangent line to the graph of  $y = f(x)$  at the point with  $x = 6$ .

$$f'(x) = \cos\left(\frac{\pi}{\sin\left(\frac{\pi}{x}\right)}\right) \cdot \frac{-\pi}{\sin^2\left(\frac{\pi}{x}\right)} \cdot \cos\left(\frac{\pi}{x}\right) \cdot \frac{-\pi}{x^2}$$

$$f'(6) = \underbrace{\cos\left(\frac{\pi}{\sin\left(\frac{\pi}{6}\right)}\right)}_{\cos\left(\frac{\pi}{1/2}\right)} \cdot \underbrace{\frac{-\pi}{\sin^2\left(\frac{\pi}{6}\right)}}_{\frac{-\pi}{(1/2)^2}} \cdot \underbrace{\cos\left(\frac{\pi}{6}\right)}_{\frac{\sqrt{3}}{2}} \cdot \underbrace{\frac{-\pi}{6^2}}_{\frac{-\pi}{6^2}} = \frac{\pi^2}{6\sqrt{3}}$$

$$f(6) = \underbrace{\sin\left(\frac{\pi}{\sin\left(\frac{\pi}{6}\right)}\right)}_{\sin\left(\frac{\pi}{1/2}\right)} = 0$$

The equation of the tangent line is

$$y - 0 = \frac{\pi^2}{6\sqrt{3}} \cdot (x - 6)$$

b. Choose one of the options by putting a  $\checkmark$  in the box to the left of it, and then explain your reasoning in one sentence.

I expect  $f(101\pi)$  to be  positive  zero  negative, because:

I expect  $\sin\left(\frac{\pi}{101\pi}\right)$  to be slightly smaller than  $\frac{1}{101}$  as the tangent line  $y=x$  to  $y=\sin x$  at the origin lies above it for  $x>0$ , and hence  $\frac{\pi}{\sin\left(\frac{\pi}{101\pi}\right)}$  to be slightly larger than  $101\pi$ .