

5. Evaluate the following integrals.

$$\begin{aligned}
 \text{a. } \int \cos 2x \cos^3 x \, dx &= \int (1-2\sin^2 x) \cos^2 x \cdot \cos x \, dx \\
 &= \int (1-2\sin^2 x)(1-\sin^2 x) \cos x \, dx \\
 &= \int (1-2u^2)(1-u^2) \, du \\
 &\quad \begin{array}{l} \uparrow \\ \boxed{u = \sin x} \\ \boxed{du = \cos x \, dx} \end{array} \\
 &= \int (1-3u^2+2u^4) \, du \\
 &= u - u^3 + \frac{2}{5}u^5 + C \\
 &= \sin x - \sin^3 x + \frac{2}{5}\sin^5 x + C
 \end{aligned}$$

$$\begin{aligned}
 \text{b. } \int_0^{\pi/4} \frac{x \sin x}{\cos^3 x} \, dx &= \int_0^{\pi/4} x \tan x \sec^2 x \, dx \\
 &= \left[x \cdot \frac{1}{2} \tan^2 x \right]_0^{\pi/4} - \frac{1}{2} \int_0^{\pi/4} \tan^2 x \, dx \\
 &\quad \begin{array}{l} \uparrow \\ \boxed{u = x \Rightarrow du = dx} \\ \boxed{dv = \tan x \sec^2 x \, dx \Rightarrow v = \frac{1}{2} \tan^2 x} \end{array} \\
 &= \frac{1}{2} \cdot \frac{\pi}{4} - \frac{1}{2} \int_0^{\pi/4} (\sec^2 x - 1) \, dx \\
 &= \frac{\pi}{8} - \frac{1}{2} \left[\tan x - x \right]_0^{\pi/4} \\
 &= \frac{\pi}{8} - \frac{1}{2} \left(1 - \frac{\pi}{4} \right) = \frac{\pi}{4} - \frac{1}{2}
 \end{aligned}$$