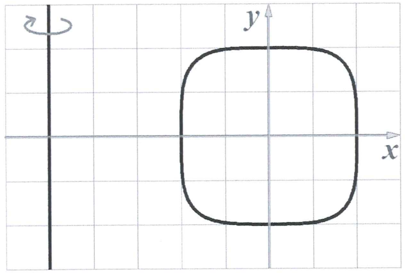
4. Let V be the volume of the water-dropper shown in the figure which has the shape obtained by revolving the curve $x^4 + y^4 = 1$ about the line $x = -5/2$ where all units are in centimeters.

a. Express V as an integral using the cylindrical shells method.

$$V = 2\pi \int_{\boxed{-1}}^{\boxed{1}} \boxed{\left(x+\frac{5}{2}\right)\cdot\left((1-x^4)^{1/4} - \left(-(1-x^4)^{1/4}\right)\right)}\, dx$$

b. Express V as an integral using the washer method.

$$V = \pi \int_{\boxed{-1}}^{\boxed{1}} \boxed{\left(\frac{5}{2} + (1-y^4)^{1/4}\right)^2 - \left(\frac{5}{2} - (1-y^4)^{1/4}\right)^2}\, dy$$

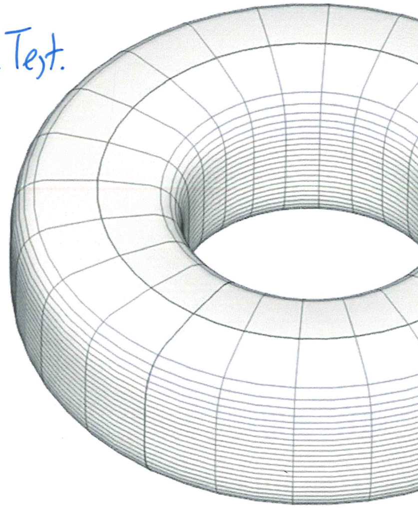


c. Show that the improper integral $\int_0^1 u^{-3/4}(1-u)^{1/4}\, du$ converges.

$0 < u \le 1 \Rightarrow 0 \le 1-u < 1 \Rightarrow 0 \le (1-u)^{1/4} < 1 \Rightarrow 0 \le u^{-3/4}(1-u)^{1/4} < u^{-3/4}$ for $0 < u \le 1$

$\int_0^1 u^{-3/4}\, du$ converges as $p = \frac{3}{4} < 1$

$\Rightarrow \int_0^1 u^{-3/4}(1-u)^{1/4}\, du$ converges by Comparison Test.



d. Express V in terms of $A = \int_0^1 u^{-3/4}(1-u)^{1/4}\, du$.

$$V = 10\pi \int_{-1}^{1} (1-y^4)^{1/4}\, dy \underset{\text{by symmetry}}{=} 20\pi \int_0^1 (1-y^4)^{1/4}\, dy = 20\pi \int_0^1 (1-u)^{1/4} \cdot \frac{1}{4} u^{-3/4}\, du$$

$$= 5\pi \int_0^1 u^{-3/4}(1-u)^{1/4}\, du = 5\pi A$$

$u = y^4$

$y = u^{1/4} \Rightarrow dy = \frac{1}{4} u^{-3/4}\, du$