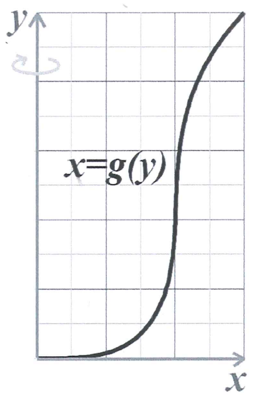
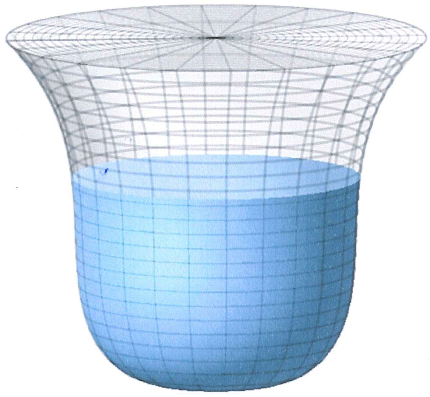
3. The side surface of a water tank has the shape generated by revolving the graph of a continuous nonnegative function $x = g(y)$ for $0 \leq y \leq 5$ with $g(0) = 0$ and $g(5) = 3$ about the y-axis where all units are in meters. Assume that:



① As water runs out of a small hole at the bottom of tank, the speed of the water flowing through the hole at any moment is proportional to the square root of the depth of the water in the tank at that moment.

② The function g is chosen in such a way that the depth of the water changes at a constant rate at all times.

Find the volume of the tank.



Let $V(h)$ denote the volume of the water when the depth is h.

$$V(h) = \pi \int_0^h g(y)^2\,dy$$

$$\Downarrow$$

$$-k_1\sqrt{h} \overset{①}{=} \frac{dV}{dt} \overset{\text{FTC1}}{=} \pi g(h)^2 \frac{dh}{dt} \overset{②}{=} \pi g(h)^2 \cdot (-k_2)$$ where k_1 and k_2 are positive constants.

$$\Downarrow$$

$$g(h) = k \cdot h^{1/4}$$ where k is a positive constant.

$$\Downarrow$$

$$3 = g(5) = k \cdot 5^{1/4}$$

$$\rightarrow g(h) = \frac{3}{5^{1/4}} \cdot h^{1/4}$$

$$\Downarrow$$

The volume of the tank $= V(5) = \pi \int_0^5 g(y)^2\,dy = \pi \int_0^5 \frac{3^2}{5^{1/2}}\, y^{1/2}\,dy = \pi \cdot \left[\frac{3^2}{5^{1/2}} \cdot \frac{y^{3/2}}{3/2}\right]_0^5 = 30\pi \text{ m}^3$