2. Consider the function $f(x) = \dfrac{\sin(\pi x^3)}{x^2+1}$.

a. Find an equation for the tangent line to the graph of $y = f(x)$ at the point with $x = 1$.

$$f'(x) = \frac{\cos(\pi x^3)\cdot \pi \cdot 3x^2 \cdot (x^2+1) - \sin(\pi x^3)\cdot 2x}{(x^2+1)^2}$$

$$f'(1) = \frac{\cos(\pi)\cdot \pi \cdot 3 \cdot 2 - \sin(\pi)\cdot 2}{2^2} = \frac{-1\cdot 6\pi - 0\cdot 2}{4} = -\frac{3\pi}{2}$$

$$f(1) = \frac{\sin(\pi)}{2} = 0$$

An equation for the tangent line is:

$$y - 0 = -\frac{3\pi}{2}\cdot(x-1)$$

b. Show that the equation $f(x) = \dfrac{3}{5}$ has at least two solutions.

$$f(0) = 0 < \frac{3}{5}$$

$$f(1) = 0 < \frac{3}{5}$$

$$f\left(\frac{1}{\sqrt[3]{2}}\right) = \frac{1}{2^{-2/3}+1} > \frac{3}{5}$$

because $32 > 27 \Rightarrow 2\cdot 2^{2/3} > 3 \Rightarrow \frac{2}{3} > 2^{-2/3} \Rightarrow \frac{5}{3} > 2^{-2/3}+1 \Rightarrow \frac{1}{2^{-2/3}+1} > \frac{3}{5}$.

f is continuous everywhere.

Hence, by IVT, there are c_1 in $\left(0, \frac{1}{\sqrt[3]{2}}\right)$ and c_2 in $\left(\frac{1}{\sqrt[3]{2}}, 1\right)$ such that $f(c_1) = \frac{3}{5}$ and $f(c_2) = \frac{3}{5}$.