**3a.** The area of the region lying above the parabola $y = x^2$ and below a line passing through the origin is 1 square unit. Find all possible values of the slope of this line.

$\left.\begin{matrix} y = mx \\ y = x^2 \end{matrix}\right\} \Rightarrow mx = x^2 \Rightarrow x = 0 \text{ or } x = m$

If $m > 0$, then: $\text{Area} = \int_0^m (mx - x^2)\,dx = \left[\frac{1}{2}mx^2 - \frac{1}{3}x^3\right]_0^m = \frac{1}{2}m^3 - \frac{1}{3}m^3 = \frac{1}{6}m^3$

So, $\text{Area} = 1 \Rightarrow m^3 = 6 \Rightarrow m = \sqrt[3]{6}$

If $m < 0$, then by symmetry $m = -\sqrt[3]{6}$.

**3b.** Find the volume of the solid generated by revolving the region between the graph of $y = 1/(1+e^x)$ and the $x$-axis for $x \geq 0$ about the $x$-axis.

$$\text{Volume} = \pi \int_0^\infty (\text{radius})^2\,dx = \pi \int_0^\infty \frac{1}{(1+e^x)^2}\,dx = \pi \int_0^\infty \frac{e^{-2x}}{(e^{-x}+1)^2}\,dx$$

$$= \pi \int_2^1 \frac{u-1}{u^2}\cdot(-du) = \pi \int_1^2 \left(\frac{1}{u} - \frac{1}{u^2}\right)du = \pi\left[\ln|u| + \frac{1}{u}\right]_1^2$$

$u = e^{-x} + 1$
$du = -e^{-x}\,dx$

$$= \pi\left(\ln 2 + \frac{1}{2} - \ln 1 - 1\right) = \pi\cdot\left(\ln 2 - \frac{1}{2}\right)$$