2a. Evaluate the integral $\int \frac{dx}{(\tan x + \cot x)^2}$.

$$\int \frac{dx}{(\tan x+\cot x)^2} = \int \frac{dx}{\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)^2} = \int \frac{\sin^2 x \cos^2 x}{(\sin^2 x + \cos^2 x)^2}\, dx$$

$$= \frac{1}{4}\int \sin^2 2x \, dx = \frac{1}{4}\int \frac{1-\cos 4x}{2}\, dx = \frac{1}{8}x - \frac{1}{32}\sin 4x + C$$

2b. A function f with a continuous derivative satisfies:

$$f(0) = 1, \qquad f(\pi/4) = 2, \qquad \int_0^{\pi/4} f(x)\, dx = 3, \qquad \int_0^{\pi/4} f(x)\tan^2 x\, dx = 4$$

Evaluate $\int_0^{\pi/4} f'(x)\tan x\, dx$.

$$\int_0^{\pi/4} f'(x)\tan x\, dx = f(x)\tan x\Big]_0^{\pi/4} - \int_0^{\pi/4} f(x)\sec^2 x\, dx$$

$u = \tan x \Rightarrow du = \sec^2 x\, dx$

$dv = f'(x)\,dx \Rightarrow v = f(x)$

$$= f\left(\tfrac{\pi}{4}\right)\tan\left(\tfrac{\pi}{4}\right) - f(0)\tan(0) - \int_0^{\pi/4} f(x)(\tan^2 x + 1)\, dx$$

$$= 2 - \int_0^{\pi/4} f(x)\tan^2 x\, dx - \int_0^{\pi/4} f(x)\, dx = 2 - 4 - 3 = -5$$