

MATH 112-04 QUIZ 5

SURNAME / NAME:

ID:

Date: March 21, 2005

Example 1. Determine whether $\int_0^{\infty} \frac{dx}{(x^3 + \arctan x)^{1/2}}$ converges.

Solution. We have $\int_0^{\infty} \frac{dx}{(x^3 + \arctan x)^{1/2}} = \int_0^1 \frac{dx}{(x^3 + \arctan x)^{1/2}} + \int_1^{\infty} \frac{dx}{(x^3 + \arctan x)^{1/2}} = I_1 + I_2$.

Step 1: Investigate whether I_1 converges. $\frac{1}{\sqrt{x^3 + \arctan x}}$ is a continuous function on $(0, 1]$ and

$$\lim_{x \rightarrow 0} \frac{\frac{1}{(x^3 + \arctan x)^{1/2}}}{\frac{1}{x^{1/2}}} = \lim_{x \rightarrow 0} \frac{1}{(x^2 + \frac{\arctan x}{x})^{1/2}} = 1, \quad (*)$$

since by H'Lopital's Rule $\lim_{x \rightarrow 0} \frac{\arctan x}{x} = \lim_{x \rightarrow 0} \frac{\frac{1}{1+x^2}}{\frac{1}{1}} = 1$. Since $\int_0^1 \frac{dx}{x^{1/2}}$ converges and $(*)$ holds,

then by the Limit Comparison Test(LCT), $I_1 = \int_0^1 \frac{dx}{(x^3 + \arctan x)^{1/2}}$ converges.

Step 2: Investigate whether I_2 converges. On $[1, \infty)$, $\sqrt{x^3 + \arctan x} > \sqrt{x^3}$, so $\frac{1}{(x^3 + \arctan x)^{1/2}} < \frac{1}{x^{3/2}}$ for any x on $[1, \infty)$.

Since $\int_1^{\infty} \frac{dx}{x^{3/2}}$ converges and $\frac{1}{(x^3 + \arctan x)^{1/2}} < \frac{1}{x^{3/2}}$ for any x on $[1, \infty)$, then by the Direct

Comparison Test(DCT), $I_2 = \int_1^{\infty} \frac{dx}{(x^3 + \arctan x)^{1/2}}$ converges.

Therefore, $\int_0^{\infty} \frac{dx}{(x^3 + \arctan x)^{1/2}} = I_1 + I_2$ converges.

Problem 1. Determine whether $\int_0^{\infty} \frac{dx}{(\sin^2 x + x^4)^{1/3}(1 + e^x)}$ converges.

Hint: You may use that $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$ and $\int_1^{\infty} \frac{1}{e^x} dx$ converges.