Solution to MATH 112-04 QUIZ 4

Problem 1. Evaluate the following integrals

$$(a)\quad \int \frac{xdx}{(8-2x^2-x^4)^{3/2}}\ , \qquad (b)\quad \int \frac{\sec^4(\ln x)\tan^5(\ln x)}{x}dx\ , \qquad (c)\quad \int_0^1 x\tan^{-1}xdx\ .$$

Solution.

$$(a)\quad \int \frac{xdx}{(8-2x^2-x^4)^{3/2}} = \int \frac{xdx}{(9-(x^2+1)^2)^{3/2}} = \frac{1}{2}\int \frac{du}{(9-u^2)^{3/2}} = \frac{1}{2}\int \frac{3\cos\theta d\theta}{27cos^3\theta}$$

$x^2+1=u$, $xdx=\frac{1}{2}du$ $\qquad$ $u=3\sin\theta,$ $du=3\cos\theta d\theta,$

$$=\frac{1}{18}\tan\theta + C = \frac{1}{18}\frac{u}{\sqrt{9-u^2}} + C = \frac{1}{18}\frac{x^2+1}{\sqrt{8-2x^2-x^4}} + C$$

$$(b)\quad \int \frac{\sec^4(\ln x)\tan^5(\ln x)}{x}dx = \int \sec^4 u\tan^5 udu = \int \sec^3 u\tan^4 u(\sec u\tan u)du =$$

$u=\ln x, du=\frac{1}{x}dx$ $\qquad$ $t=\sec u, dt=\sec u\tan udu$, $\tan^4 u=(\tan^2 u)^2=(\sec^2 u-1)^2$

$$=\int t^3(t^2-1)^2dt = \int(t^7-2t^5+t^3)dt = \frac{t^8}{8}-\frac{t^6}{3}+\frac{t^4}{4}+C = \frac{\sec^8(\ln x)}{8}-\frac{\sec^6(\ln x)}{3}+\frac{\sec^4(\ln x)}{4}+C$$

$$(c)\quad \int_0^1 x\tan^{-1}xdx = \frac{x^2}{2}\tan^{-1}x\Big|_0^1 - \frac{1}{2}\int_0^1\frac{x^2}{x^2+1}dx = \frac{\pi}{8}-\frac{1}{2}\int_0^1\frac{x^2+1-1}{x^2+1}dx =$$

$u=\tan^{-1}x, xdx=dv$
$v=\frac{x^2}{2}, vdu=\frac{1}{2}\frac{x^2}{1+x^2}$

$$=\frac{\pi}{8}-\frac{1}{2}\int_0^1 dx+\frac{1}{2}\int_0^1\frac{dx}{x^2+1} = \frac{\pi}{8}+(-\frac{1}{2}x+\frac{1}{2}\tan^{-1}x)\Big|_0^1 = \frac{\pi}{4}-\frac{1}{2}$$