

Due on: March 15, 2006.

MATH 110 Homework 1

Problem 1. Construct a truth table for each of the following compound propositions.

(a) $p \wedge (\bar{q} \vee p)$;

(b) $\overline{p \wedge (q \vee p)} \longleftrightarrow p$.

Problem 2. Prove that

(a) $1^2 - 2^2 + 3^2 - 4^2 + \dots + (-1)^{n-1}n^2 = (-1)^{n-1} \frac{n(n+1)}{2}, \forall n \geq 1$;

(b) $5^{2n} - 2^{5n}$ is divisible by 7, $\forall n \geq 1$;

(c) $\sum_{i=1}^n (i+1)2^i = n2^{n+1}, \forall n \geq 1$.

Problem 3. A student must answer exactly 8 questions out of 10 questions in an examination.

(a) How many choices does the student have?

(b) How many choices does the student have if he/she must answer at least four of the first five questions?

Problem 4. There are 20 varieties of chocolates available and Selin wants to buy eight chocolates.

(a) How many choices does she have if her boyfriend insists that at least one chocolate should have a cherry center? (At least 8 chocolates of all varieties are available.)

(b) How many choices does she have if there remain only two chocolates with caramel centers? (At least 8 chocolates of all other varieties are available.)

Problem 5. Let $R = \{(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 2), (3, 3), (4, 4)\}$ be a binary relation on the set $A = \{1, 2, 3, 4\}$. Is R reflexive? Symmetric? Transitive? Explain your answers.

Problem 6. (a) Let R be a symmetric and transitive relation on a set A . Show that if for every a in A there exists b in A such that (a, b) is in R then R is an equivalence relation.

(b) Let R be a reflexive and transitive relation on A . Let T be a binary relation on A such that (a, b) is in T if and only if both (a, b) and (b, a) are in R . Show that T is transitive.

Problem 7. If 9 red balloons and 6 blue balloons are to be distributed to four children, how many ways are possible if every child must receive a balloon of each color?

Problem 8. Call a relation R on a set A "circular" if

$$(x, y) \in R \text{ and } (y, z) \in R \text{ imply } (z, x) \in R.$$

Prove that R is an equivalence relation if and only if R is both reflexive and circular.

Problem 9. Let $A = \{1, 2, 3, 4, 5\} \times \{1, 2, 3, 4, 5\}$ and $R = \{((x_1, y_1), (x_2, y_2)) : x_1 + y_1 = x_2 + y_2\}$.

(a) Verify that R is an equivalence relation on A .

(b) Determine the partition of A induced by R .

Problem 10. Consider the following advertisement for a game:

(a) There are three statements in this advertisement.

(b) Two of them are not true.

(c) This game is interesting.

Is statement (c) true? Explain your answer.