Due on: April 27, 2006.

MATH 110 Homework 2

Problem 1. Alp has promised to increase his vocabulary by learning the meaning of 90 new words during his summer holidays. Suppose he has 53 days in which he accomplish this task and will learn at least one new word a day. Show that during some span of consecutive days, Alp will learn precisely 15 new words.

Problem 2. If G is a graph on n vertices, $n \geq 2$, and G is not connected, prove that $\overline{G}$ is connected.

Problem 3. The circumference of a "roulette wheel" is divided into 36 sectors to which numbers 1,2,3, ... ,36 are assigned in some arbitrary manner. Show that there are three consecutive sectors such that the sum of their assigned numbers is at least 56.

Problem 4. (a) Explain why it is not possible to draw a loop-free connected undirected graph with eight vertices where the degrees of the vertices are 1,1,1,2,3,4,5,7.
(b) Give an example of a loop-free connected undirected multigraph with eight vertices where the degrees of the vertices are 1,1,1,2,3,4,5,7.

Problem 5. Use the Principle of Mathematical Induction to prove the following statement. If the graph $G = (V, E)$ has even number of vertices and does not have cycles of length 3 than $|E| \leq \dfrac{|V|^2}{4}$.
Hint: Prove by the Principle of Mathematical Induction on n, where $|V| = 2n$.

Problem 6. Let $G = (V, E)$ be a loop-free undirected graph with five vertices of degree 5, three vertices of degree 4, one vertex of degree 3. How many edges does graph G have? How many edges does graph $\bar{G}$ have?

Problem 7. Find the number of different subgraphs $H = (V, E)$ of graph K_8 that have exactly 13 edges.

Problem 8. 8 cities are connected by a network of 23 highways. (A highway is defined to be a road between two cities that does not go through any intermediate cities.) Show that one can always travel between any two city through connecting highways.

Problem 9. Suppose G_1 and G_2 are two graphs that have Euler circuits. Suppose G_1 and G_2 do not have vertices in common. Let v_1 be a vertex in G_1 and let v_2 be a vertex in G_2. Join v_1 and v_2 with a single edge. Does a resulting graph possess an Euler circuit? Explain.

Problem 10. Show that if any vertex (and the edges incident to it) is removed from the Petersen graph, then the resulting graph has a Hamiltonian cycle.