MATH 110 FINAL EXAM

Date: May 18, 2005, Time: 9:00-11:00

SURNAME/NAME:...

ID:.................................. Section................

1 Check that there are 5 questions on your booklet.

2 Show all your work. Correct answers without sufficient explanation may not get full credit.

1	2	3	4	5	TOTAL
15	25	20	20	20	100

Problem 1. (15 points) Among the many rooms in an old house, there is a ghost in each room that has an even number of doors. If the house has only one entrance, prove that a person entering from outside can always reach a room in which there is no ghost.

Problem 2. (a)(15 points)For

$$A = \{(-4,-20), (-3,-9), (-2,-4), (-1,-11), (-1,-3), (1,2), (1,5), (2,10), (2,14), (3,6), (4,8), (4,12)\}$$

define the relation R on A as follows:

$$R = \{((a,b),(c,d)) \quad | \quad ad = bc\}.$$

Is R an equivalence relation on A? Explain. If yes, find the equivalence classes $[(2,14)]$ and $[(-3,-9)]$.

(b) (10 points) If $|B| = 30$ and the equivalence relation R on B partitions B into three disjoint equivalence classes B_1, B_2, and B_3, where $|B_1| = |B_2| = |B_3|$, what is $|R|$?

Problem 3. (20 points) Let S be a set of five positive integers the maximum of which is at most 9. Prove that the sum of the elements in all the nonempty subsets of S cannot be distinct.

Problem 4. (a) (10 points) Let f and g be homomorphisms from a group $(G, \circ)$ to a group $(H, \star)$. Is

$$K = \{x \in G \quad | \quad f(x) = g(x)\}$$

a subgroup of G? Explain.

(b) (10 points) Let $(G, \circ)$ be a group that consists of 8 elements, i.e. $|G| = 8$. Show that there is an element $a \in G$ such that $a \circ a = e$, where e is the identity element in G and $a \neq e$.

Problem 5. (a) (10 points) Let H and K be subgroups of a group G. Show that if $|H| = 10$ and $|K| = 21$, then $H \cap K = \{e\}$, where e is the identity element of G.
(b) (10 points) Let $(G, \circ)$ be a cyclic group that consists of 7 elements, i.e. $|G| = 7$. How many distinct generators does G have? Explain.