

Solution to Problem 9. (See Solution to Problem 3 from Quiz 5, sec.

Solution to Problem 10 (a) By Lagrange's Theorem, every

subgroup H of G has either 2 or p elements.

If $|H|=2$, then $H = \{e, a\} = \langle a \rangle$ is a cyclic group generated by a .

If $|H|=p$, then any element in H except e has order p .

Take $h \in H, h \neq e$. Then $\text{ord}(h) = p$. Hence

$|\langle h \rangle| = |H|$ and then $\langle h \rangle = H$. It implies H is a cyclic subgroup generated by h .

(b) Let $|G| = p^2$. Then $\forall h \in G, h \neq e$, order of h is either p or p^2 (by Lagrange's Theorem). Take any $h \neq e$. If order of h is p then we found an element of order p and there is nothing to prove.

If order of h is p^2 , consider $a = \underbrace{h^p}_{p \text{ times}} = \underbrace{h \circ h \circ \dots \circ h}_p$

Then $a \neq e$ and $a^p = (h^p)^p = e$, i.e.

order of a is p . Again, we found an element of order p .