Solution to Problem 5. Since G is cyclic and $|G|=6$ then G is isomorphic to $(\mathbb{Z}_6,+)$. In $(\mathbb{Z}_6,+)$ we have only two generators — elements $[1]$ and $[5]$. Then, in G we also have only two generators.

Remark: Let $T:\mathbb{Z}_6 \to G$ be isomorphism then $T([1])$ and $T([5])$ have orders 6 and hence generate group G.

Solution to Problem 6. Let us prove this property by induction on m. For $m=2$, $T(a_1 \circ a_2) = T(a_1) * T(a_2)$ because T is a homomorphism. (Base holds)

Assume
$$T(a_1 \circ a_2 \circ \dots \circ a_m) = T(a_1) * T(a_2) * \dots * T(a_m)$$
for some $m \geq 2$. Consider Induction Step $m \rightsquigarrow m+1$:
$$T(a_1 \circ a_2 \circ \dots \circ a_m \circ a_{m+1}) \underset{\substack{\uparrow \\ T \text{ is homomorphism}}}{=} T(a_1 \circ a_2 \circ \dots \circ a_m) * T(a_{m+1}) \underset{\substack{\uparrow \\ \text{induction} \\ \text{assumption}}}{=}$$
$$= T(a_1) * T(a_2) * \dots * T(a_m) * T(a_{m+1}) \quad \square \quad \text{(Induction Step holds)}$$

Solution to Problem 7. (See Solution of Problem 3 from Quiz 5, section 2).

Solution to Problem 8. We have to show that T is a homomorphism, one-to-one and onto.

Homomorphism: $T(x \circ y) = (x \circ y)^2 = x \circ y \circ x \circ y \underset{\substack{\uparrow \\ G \text{ is commutative}}}{=} x \circ x \circ y \circ y = x^2 \circ y^2 =$
$$= T(x) \circ T(y) \Rightarrow T \text{ is a homomorphism.}$$

One-to-one: $T(x) = T(y) \Leftrightarrow x^2 = y^2$. Let $|G| = 2n+1$. Then
$$x^{2(n+1)} = y^{2(n+1)}, \text{ i.e. } x^{2n+2} = y^{2n+2}, \text{ i.e.}$$
$$x^{(2n+1)} \circ x = y^{2n+1} \circ y, \text{ i.e. } x = y$$
(By Corollary 1 to Lagrange's Theorem, $x^{2n+1} = e$, $y^{2n+1} = e$)
Since $T(x) = T(y)$ implies $x = y$ then T is one-to-one.

Onto: We have to show that for any $y \in G$ $\exists x \in G$ such that $T(x) = y$, i.e. $x^2 = y$.
For a given $y \in G$ consider $x = y^{n+1}$. Then
$T(x) = T(y^{n+1}) = y^{2n+2} = y^{2n+1} \circ y = e \circ y = y$.
Then T is onto.