MATH 116 QUIZ 4

Name : **Student Id :**

Section :

Problem: Let function $z = f(x, y)$ be defined by the equation

$$e^{x+z} + y^2 + z = 2$$

and $f(0, 1) = 0$.

(a) Find the gradient of function $f(x, y)$ at the point $P(0, 1)$.

Solution:

- $\frac{\partial}{\partial x}(e^{x+z} + y^2 + z) = \frac{\partial}{\partial x}(2)$
  $e^{x+z}(1 + \frac{\partial z}{\partial x}) + \frac{\partial z}{\partial x} = 0$

$$\frac{\partial z}{\partial x} = \frac{-e^{x+z}}{e^{x+z} + 1} \qquad \frac{\partial z}{\partial x}|_P = \frac{-e^{0+0}}{e^{0+0} + 1} = -\frac{1}{2}$$

- $\frac{\partial}{\partial y}(e^{x+z} + y^2 + z) = \frac{\partial}{\partial x}(2)$
  $e^{x+z}\frac{\partial z}{\partial y} + 2y + \frac{\partial z}{\partial y} = 0$

$$\frac{\partial z}{\partial y} = \frac{-2y}{e^{x+z} + 1} \qquad \frac{\partial z}{\partial y}|_P = \frac{-2}{e^{0+0} + 1} = -1$$

$$\nabla f(0, 1) = -\frac{1}{2}\overrightarrow{i} - \overrightarrow{j}$$

(b) Find the linear approximation of function $f(x, y)$ at the point $P(0, 1)$.

Solution:

$$L(x, y) = f(0, 1) + f_x(0, 1)(x - 0) + f_y(0, 1)(y - 1)$$

$$L(x, y) = -\frac{1}{2}x - (y - 1) = -\frac{1}{2}x - y + 1$$