MATH 116-02 QUIZ 3

Name : **Student Id :**

Section :

Problem: Let $f(x,y,z) = xy + yz + xz$, $P_0(1,-1,2)$, $\overrightarrow{A} + 3\overrightarrow{i} + 6\overrightarrow{j} - 2\overrightarrow{k}$.

(a) Find the gradient of function $f(x,y,z)$ at the point P_0.
Solution: We have,

$$\begin{aligned} f_x &= y + z, \qquad f_x(P_0) = 1 \\ f_y &= x + z, \qquad f_y(P_0) = 3 \\ f_z &= y + x, \qquad f_z(P_0) = 0 \end{aligned}$$

Since $\nabla f|_{P_0} = f_x(P_0)\overrightarrow{i} + f_y(P_0)\overrightarrow{j} + f_z(P_0)\overrightarrow{k}$

$$\nabla f|_{P_0} = \overrightarrow{i} + 3\overrightarrow{j}$$

(b) Find the derivative of f at P_0 in the direction of $\overrightarrow{A}$.
Solution: Since $\overrightarrow{u} = \frac{\overrightarrow{A}}{\|\overrightarrow{A}\|} = \frac{3\overrightarrow{i}+6\overrightarrow{j}-2\overrightarrow{k}}{7} = \frac{3}{7}\overrightarrow{i} + \frac{6}{7}\overrightarrow{j} - \frac{2}{7}\overrightarrow{k}$

Then

$$\begin{aligned} D_{\overrightarrow{u}} f(P_0) &= \nabla f|_{P_0} \cdot \overrightarrow{u} \\ &= (\overrightarrow{i} + 3\overrightarrow{j} + 0\overrightarrow{k}) \cdot (\tfrac{3}{7}\overrightarrow{i} + \tfrac{6}{7}\overrightarrow{j} - \tfrac{2}{7}\overrightarrow{k}) \\ &= \tfrac{3}{7} + \tfrac{18}{7} = 3, \end{aligned}$$

That is $D_{\overrightarrow{u}} f(P_0) = 3.$

(c) Find an equation for the tangent plane to the surface $f(x,y,z) = -1$ at the point P_0.
Solution:
The equation of the tangent plane to $f(x,y,z) = \text{constant}$ at P_0 is

$$f_x(P_0)(x - x_0) + f_y(P_0)(y - y_0) + f_z(P_0)(z - z_0) = 0$$

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In our case, the tangent plane is

$$(x - 1) + 3(y + 1) + 0(z - 2) = 0$$

$$x + 3y + 2 = 0$$